CHAPTER – I: UNITS & MEASUREMENT

Physics is the most basic science. Scientists in every field of science use physics, just as chemists use physics to study the internal structure of atoms, archaeologists to study how dinosaurs walked, meteorologists to study how human activities affect the oceans and weather. Physics is the foundation of engineering and technology. No engineer can make a flat TV without knowledge of physics. Not only that, you can't even make a rat cage without knowing the basic principles of physics.

Physics is an experimental science. Physicists observe phenomena occurring in nature and any specifics in themPattern tries to find relation with this occurring phenomenon. These patterns are called physical theories and when they are well established and widely used, these physical theories are called laws of physics.

In order to formulate physical theories, they are first tested by experiments. For this, experimental observations are taken. Figures are needed to represent the results of these experiments.

The numerals required for the numerical or quantitative description of any physical phenomenon are the physical signs (Physical Quantity) is called. As your external description requires two physical signs called "weight" and "height".

A reference to measure a physical sign(Reference) is required. For example, your height is 1.5 meters, which means you are 1.5 times taller than the meter scale. Such reference defines those entities.

When we use numerals to describe a physical sign, we also have to specify the unit of that numeral. Just because of your heightWriting “1.5” is meaningless for physics.
Accurate and reliable measurements in physics require units of measurement that are unchanging and easily replicated.canIt should be like that.

A standardized measure of a physical quantity is called a unit of that physical quantity.

Features of Units:

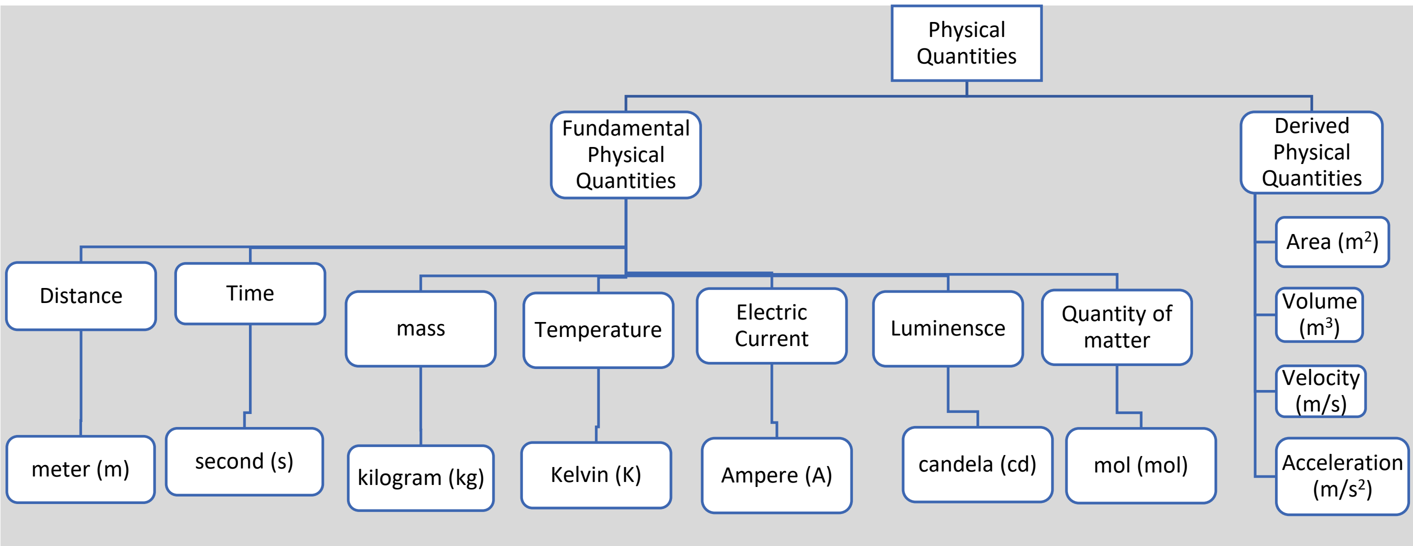
1. The size of the unit should be fixed and clear.
2. The size of the unit must be immutable and the event defining it (if any) must be permanent.
3. Units should be easily replicable and easily obtainable.

Physical Quantities units and unit systems:

Determining the units of some minimal physical quantities among the many physical quantities found in practice can determine the units of all the remaining physical quantities.

Basic physical quantities(Fundamental Physical Quantities): The physical quantities which do not depend on other physical quantities i.e. the quantities which have independent existence are called fundamental physical quantities.

Derived Physical Quantities: The physical quantities are derived from the fundamental physical quantities or depend on fundamental physical quantities are called Derived Physical quantities.



Different methods have been implemented for the measurement of physical signs.

1. CGS Method (Centimeter, Gram, Second)
2. MKS Method (Meters, Kilograms, Seconds)
3. FPS method (feet, pound, second)
4. SI System (Metric System)

SI Method: In 1971, Institute at Paris in France "Under the auspices of the International Bureau of Weights and Measures, the 14th meeting of the General Conference on Weights and Measures adopted an international system of units known as the SI system of units. Seven physical quantities are accepted as Fundamental physical quantities in the SI system. Following are the Fundamental physical quantities of the SI system, their symbols and their SI units.

Fundamental Physical Quantity	Units	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	Kelvin	K
Electric Current	Ampere	A
Luminescence or Luminosity	candela	cd
Quantity of Matter	mol	mol

Derived Physical Quantities: The physical quantities are derived from the fundamental physical quantities or depend on fundamental physical quantities are called Derived Physical quantities.

Sr. No.	Physical Quantity	Symbol	Formula	SI unit
1	Area	A	$length \times width$	$\mathrm{m^2}$
2	Volume	V	$length \times width \times height$	$\mathrm{m^3}$
3	Velocity	v	$\frac{Distance}{time}$	$\mathrm{m/s}$
4	Acceleration	a	$\frac{Velocity}{time}$	$\mathrm{m/s^2}$
5	Force	F	$mass \times acceleration$	$\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$ or N
6	Work done	W	$Force \times distance$	$\mathrm{N \cdot m}$ or J
7	Power	P	$\frac{Work\ done}{Time}$	$\mathrm{J/s}$ or W
8	Density	ρ	$\frac{mass}{volume}$	$\mathrm{kg/m^3}$
9	Surface Tension	T	$\frac{Force}{length}$	$\mathrm{N/m}$
10	Pressure	P	$\frac{Force}{Area}$	$\mathrm{N/m^2}$
11	Momentum	p	$mass \times velocity$	$\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$
12	Periodic Time	T	$Time$	s
13	Frequency	f	$\frac{1}{Periodic\ Time}$	$\mathrm{s^{-1}}$ or Hz

Prefix in SI system: This multiple or submultipleis in power of 10 form.

Prefixes	Symbol	SI	Prefixes	Symbol	SI
Yotta	Y	10^{24}	Deci	d	10^{-1}
Zetta	Z	10^{21}	centi	c	10^{-2}
Exa	E	10^{18}	Milli	m	10^{-3}
Peta	P	10^{15}	micro	μ	10^{-6}
Tera	T	10^{12}	Nano	n	10^{-9}
Giga	G	10^{9}	Pico	p	10^{-12}

Mega	M	10^6	Femto	f	10^{-15}
Kilo	k	10^3	Atto	a	10^{-18}
Hecto	h	10^2	Zepto	z	10^{-21}
Deca	da	10^1	Yocto	y	10^{-24}
meter	m	$10^0 = 1$			

Conversion: MKS $\rightleftharpoons$ CGS

1. Prove: $\mathbf{1\ N = 10^5\ dyne}$

   According to Newton's second law of motion,

$$F = ma$$
$$[F] = N = kg \cdot \frac{m}{s^2}$$
$$1\ \mathrm{N} = 1\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$$

We know, $1\ \mathrm{kg} = 10^3\mathrm{g}$ & $1\ \mathrm{m} = 10^2\ \mathrm{cm}$,

$$1\ \mathrm{N} = \frac{10^3 * 10^2}{1}\frac{\mathrm{g \cdot cm}}{\mathrm{s^2}}$$
$$\mathrm{dyne} = \frac{\mathrm{g \cdot cm}}{\mathrm{s^2}}$$
$$1\ \mathrm{N} = 10^5\frac{\mathrm{g \cdot cm}}{\mathrm{s^2}}$$
$$\mathbf{1\ N = 10^5\ dyne}$$

2. Prove: $\mathbf{1\ Joule = 10^7\ erg}$

   According to equation of Energy (K.E. or P.E.),

$$K.E. = \frac{1}{2}mv^2$$
$$[K.E.] = J = kg \cdot \frac{m^2}{s^2}$$
$$\therefore 1\ \mathrm{J} = \mathrm{kg} \cdot \frac{m^2}{\mathrm{s^2}}$$

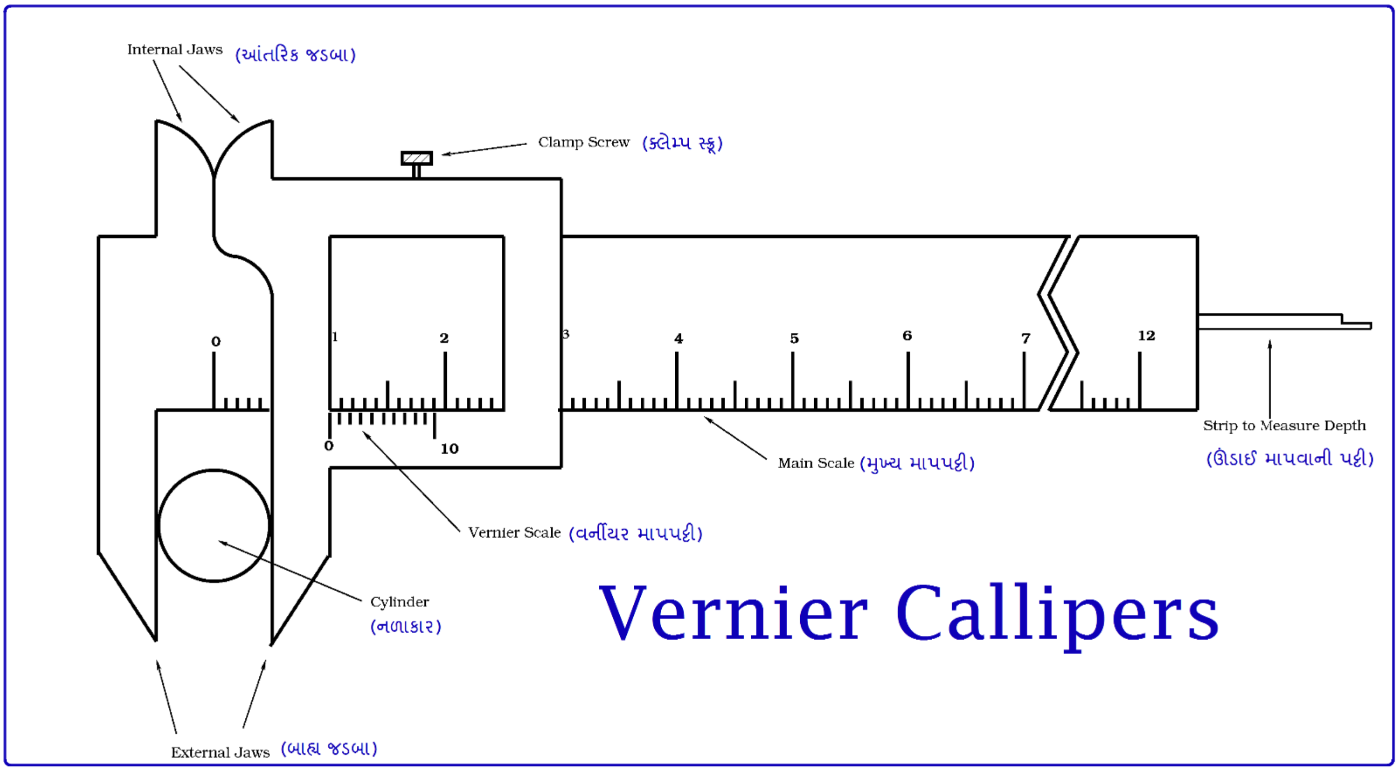
   We know, $1\ \mathrm{kg} = 10^3\mathrm{g}$ & $1\ \mathrm{m^2} = 10^4\ \mathrm{cm^2}$,

$$\therefore 1\ \mathrm{J} = 10^3\mathrm{g} \cdot \frac{10^4\ \mathrm{cm^2}}{1\ \mathrm{s^2}}$$
$$\therefore 1\ \mathrm{J} = 10^7\frac{\mathrm{g \cdot cm^2}}{\mathrm{s^2}}$$
$$\therefore 1\ \mathrm{J} = 10^7\mathrm{erg} \quad \left(\because 1\ \mathrm{erg} = \frac{\mathrm{g \cdot cm^2}}{\mathrm{s^2}}\right)$$
$$1\ \mathrm{Joule} = 10^7\ \mathrm{erg}$$

Measurement: Accurate and reliable measurement of things like length, width, height and diameter is a very important aspect in the physical world. E. g. The diameter of a very thin copper wire cannot be measured with a compass box ruler. Because only up to 1 mm can be accurately measured with this tape measure, which is called the minimum measuring power of the tape measure. So, objects which are smaller than 1 mm size cannot be measured with this caliper. Scientific instruments like vernier calipers or micrometer screw gauges are used to measure such minute distances.

Vernier Callipers: In 1631 French Mathematician P. Vernier invented the Vernier scale, with the help of which a distance of 10^{th} (or 20^{th}) of 1 mm can be measured accurately. The instrument which was designed based on this calibration is called **Vernier Calipers**.

Construction: Simple Vernier Callipers has a sliding vernier scale having n equal divisions on the main scale marked in millimeters which are kept equal to the $(n-1)$ divisions of the main scale. The main scale consists of two inner jaws and two outer jaws. The main scale has a strip for measuring depth.



Least Count: *The smallest measurement that can be accurately taken with any instrument is called least count of that instrument.*

For Vernier Callipers the least count is equal to the difference between one Main Scale Division (MSD) and one Vernier Scale Division. Usually, 10 divisions of the vernier scale equals 9 divisions of the main scale.

$$\therefore 10\ VSD = 9\ MSD$$

$$\therefore 1\ VSD = \frac{9}{10}\ MSD$$

$$Least\ Count = 1\ MSD - 1\ VSD$$

$$\therefore L.C. = 1\ MSD - \frac{9}{10}\ MSD$$

$$\therefore L.C. = \left(1 - \frac{9}{10}\right) MSD = \left(\frac{10-1}{10}\right) MSD$$

$$\therefore L.C. = \frac{1\ MSD}{10}$$

The main scale of vernier scale id calibrated into *mm*. Hence, $1\ MSD\ =\ 1\ mm$

$$\therefore L.C. = \frac{1\ mm}{10}$$

$$L.C. = 0.1\ mm = 0.01\ cm$$

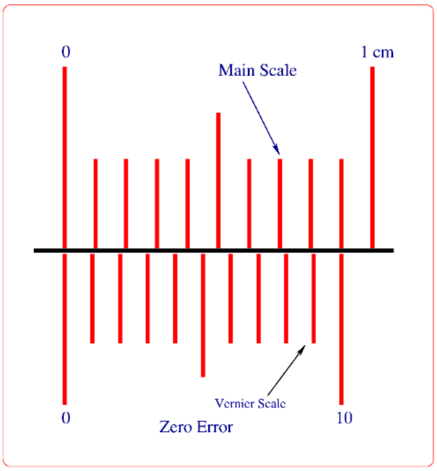
Alternatively, one can use following formula to calculate the least count of the vernier callipers.

$$L.C. = \frac{Value\ of\ smallest\ division\ of\ Main\ scale}{Total\ number\ of\ division\ of\ vernier\ scs}$$

$$\therefore L.C. = \frac{1\ mm}{10}$$

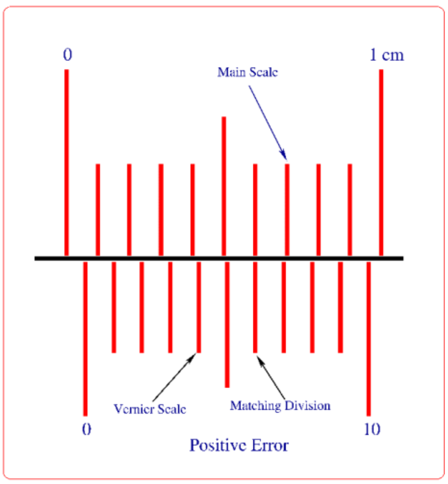
$$L.C. = 0.1\ mm = 0.01\ cm$$

In this way the length can be measured accurately to tenths (or twentieths) of mm with the help of Vernier Callipers.



Zero Error: When jaws of vernier callipers are closed, if the zeroth division of the vernier scale coincides with the zeroth division of the main scale then vernier callipers is said to have Zero Error. This is the *ideal condition*. But if the zeroth division of the vernier scale and the zeroth division of the main scale do not coincide, then two types of error arise. (i) Positive error (ii) Negative Error

Positive Error: If the zeroth division of vernier scale lies on the right-hand side of the zeroth division of main scale then, vernier callipers said to have Positive error.



Due to this error the observation taken is recorded slightly higher than the true observation, so this error should be subtracted from the recorded observation to get the correct observation.

To calculate this error, the division of the vernier which coincides with any division of main scale. The number is multiplied with least count of the vernier calipers.

$$Positive\ error\ \ e = coincding\ division\ of\ V.S.\ \times L.C.$$

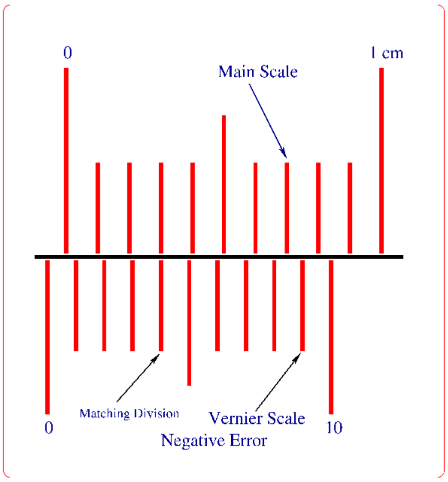
For example if the 6^{th} division of the vernier scale is coinciding with the main scale division.

$$Positive\ Error = \ 6 \times 0.01\ \text{cm} = 0.06\ \text{cm}$$
$$Positive\ Error = +0.06\ \text{cm}$$

Hence to get the correct observation $+0.06\ cm$ has to be subtracted from the observation with error.

Negative Error: If the zeroth division of vernier scale lies on the left-hand side of the zeroth division of main scale then, vernier callipers said to have **Negative error**.

Due to this error the observation taken is recorded slightly lower than the true observation, so this error should be added to the recorded observation to get the correct observation.



First, it is noted which division of the vernier is coinciding with any division of the main scale. From this negative error is calculated by following formula,

$$Negative\ Error = matching\ division\ of\ V.S. \times L.C.$$

For example if the 4th division of the vernier scale is coinciding with the main scale division.

$$Negative\ Error = 4 \times 0.01\ \text{cm} = 0.04\ \text{cm}$$
$$Negative\ Error = 0.04\ \text{cm}$$

Hence to get the correct observation $0.04\ cm$ has to be subtracted from the observation with error.

Caution !!!: **To calculate correct observation from observation with positive or negative error, learner should keep in mind that error has to be always subtracted from the taken observation, no matter it is positive error or negative error.**

$$Correct\ Observation = Taken\ observation - error$$
$$Correct\ Observation = Taken\ observation - (+Positive\ error)$$
$$Correct\ Observation = Taken\ observation - Positive\ error$$
$$Correct\ Observation = Taken\ observation - (-Negative\ error)$$
$$Correct\ Observation = Taken\ observation + Negative\ error$$

Method of Observation: First of all the least count of the of vernier calipers is calculated. Now the object of interest which is to be measured is held between the two external jaws. Then the main scale and vernier scale are observed. Suppose the zeroth division of the vernier lies between 1.3 and 1.4 cm of the main scale. Hence the observation of the main scale is recorded as 1.3 cm. Now find out of 10 divisions, which vernier scale division is exactly coinciding with the main scale division is called the observation of the vernier scale. Suppose the seventh division of the vernier scale coincides with the main scale. Now according to formula to find total observation,

$$Diameter = Main\ scale\ observation + (Vernier\ scale \times L.C)$$

$$\text{D} = 1.3 + (7 * 0.01) = 1.3 + 0.07$$

$$\text{D} = 1.37\ \text{cm}$$

Applications:

1. Thickness of thin plate
2. Depth of liquid filled in container

3. **Medical Purpose:** Measurement of medical and surgical instrument because every instrument is highly sensitive to even a little change and has to be measured precisely as the y need to function in a very sophisticated space.
4. **Steel Industries:** Manufacturing or checking the dimensions of the by-product in order to ensure the desired standards are mee, vernier callipers are highly recommended. It helps in measuring holes, width of pipes, the circumference of metal beads etc.
5. **Aerospace industries:** The aviation industry works on pure precision, a small difference in dimension of the object can be catastrophic. This device helps to measure along with precision in the use of small part which helps for flying.
6. **Science lab:** It is used for studying the expansion of material due to change in temperature. It is also used for measurement of regular and irregular shaped objects from interior or exterior.
7. **Locksmithing:** Designing safes requires part to interlock in very precise patterns and designing parts with much higher precision. Moreover, keys that are used to open locks and safes must be very precisely tooled. For this purpose, vernier calipers immensely useful in this field.

Micrometer Screw Gauge: With the help of this instrument, lengths as small as one millionth of a meter can be measured accurately.

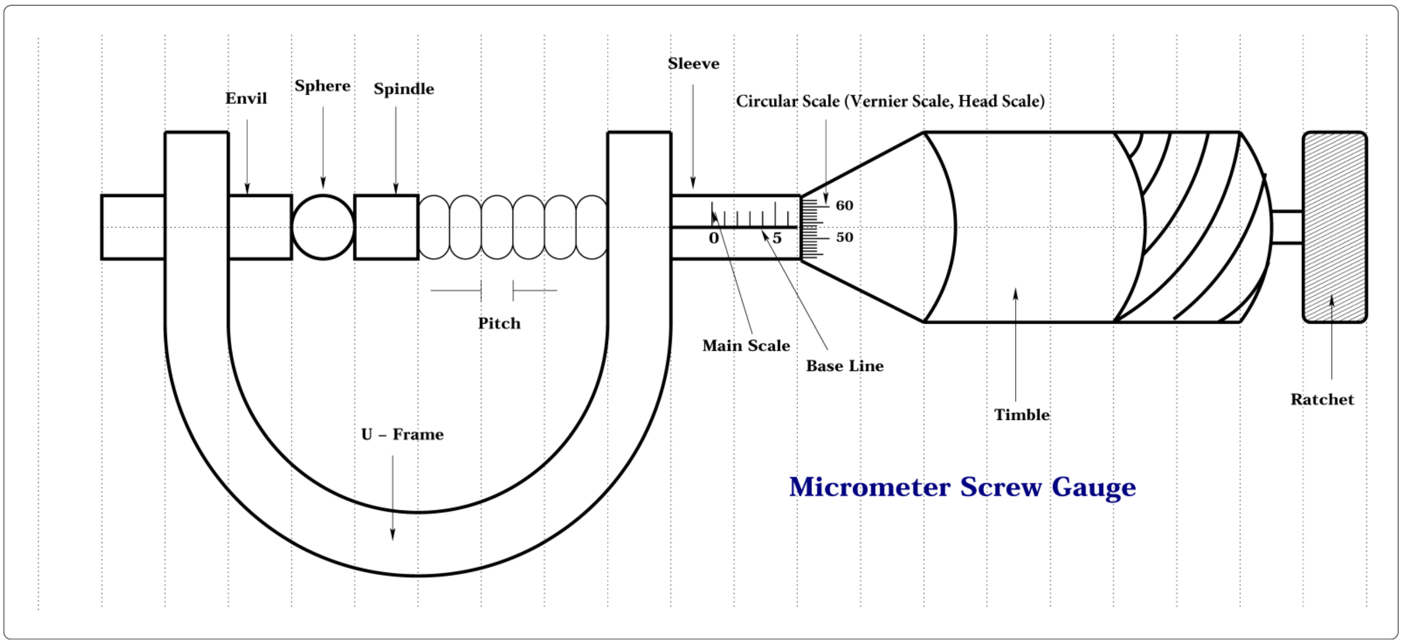
Principle: This instrument works on the principle of screw.

Construction: A micrometer has a U-shaped metal frame. One end of it is closed to the frame by a piece of metal is called stud, which is attached to an anvil. The other end is attached to a screw which can move in a circular motion. The other end of the screw is attached to a main scale marked in millimeters. Above the main scale there is a circular scale (also known as head scale) equally divided into 100 (or 50) divisions.

Least Count: A screw is designed in such a way that when a complete rotation is given, it moves a linear distance equal to one division of main scale. The distance between two successive threads of the screw is called the pitch of the screw.

$$Least\ count = \frac{Pitch}{Total\ number\ of\ divisions\ on\ circular\ scale}$$

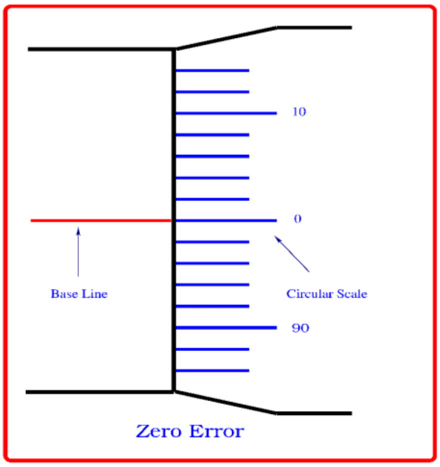
$$Least\ count = \frac{1\ mm}{100} = 0.01\ mm$$



Working: To measure the dimension of given object it is held between the anvil and spindle. Main scale observation is recorded in mm from the visible prat of Main scale. Next step is to find the circular scale division which coincides with base line (drawn on main scale). According to the formula,

$$Observation = Main\ scale\ observation + (Circular\ \ scale \times L.C)$$

Zero Error: When the anvil and spindle are brought into contact (without placing any object) if the zero division of the circular scale coincide with the Base Line of main scale, then, the instrument is said to have Zero Error. This is *ideal situation*.

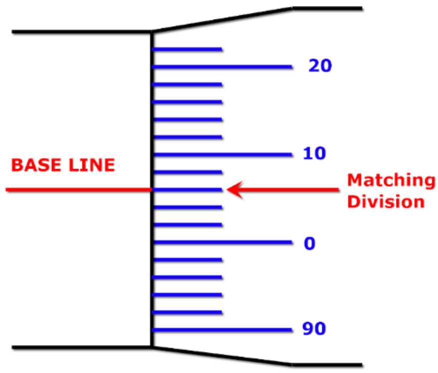


Positive Error: If the zero division of the circular scale lies below the base line, such error is called a **Positive Error**. To find this error, which division of the circular scale coincides with base line mark.

E.g. If the sixth division of the circular scale coincides with base line of the main scale,

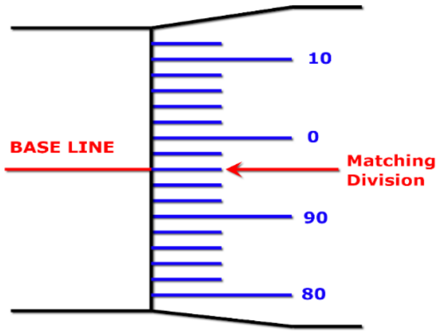
$$Positive\ Error\ \ e = 6 \times 0.01\,mm = +0.06\,mm$$

Since this error is positive, it must be subtracted from the observation recorded by the instrument to obtain the correct measurement.



Positive Error

Negative Error: If the zero division of the circular scale lies above the base line, such error is called a **Negative Error**. To find this error, which division of the circular scale coincides with base line mark.



Negative Error

E.g. If the fourth division of the circular scale coincides with base line of the main scale,

$$Negative\ Error\ \ e = 4 \times 0.01\ mm = -0.04\ mm$$

Since this error is Negative, it must be added to the observation recorded by the instrument to obtain the correct measurement.

$$\begin{aligned} L.C. &= 0.01\ mm \\ &= 10^{-2}\ mm \\ &= 10^{-2} \times 10^{-1} cm \\ &= 10^{-2} \times 10^{-1} \times 10^{-2}\ m \\ &= 10^{-5}\ m \\ &= 10 \times 10^{-6}\ m \\ &= 10\ \mu m \end{aligned}$$

thus, Least Count is of the order of micrometer and this instrument works on the principle of screw, Hence, this instrument is called *Micrometer Screw Gauge*.

Accuracy & Precision:

There is always remains uncertainty in every measurement of physical quantities. E.g. If you try to measure the thickness of this book with a simple tape, then suppose your observation is 1.7 cm. But if you take this observation with vernier calipers, your answer might be 1.76 cm. Thus, the difference between these two observations is called the uncertainty of the observations, also called the error. You can take observations to the limit of 0.1 cm with a simple caliper, while with vernier calipers this limit can be increased to 0.01 cm.

Accuracy: The closeness of the measured value of any physical quantity to standard or true value of that physical quantity is called Accuracy.

Precision: The closeness of two or more measurements to each other is known as Precision. In other wors it is how repeatable a measurement is.

Now let's take an example of measurement of a metal cube of side 2.555 cm each. Suppose you take measurement with vernier callipers of least count 0.01 cm and it comes out to be 2.51 cm. Now if you take same measurement with Micrometer screw gauge with least count of 0.001 cm and it comes out to be 2.748 cm.

Then out of two cases, the measured value 2.51 cm very close to 2.555 cm than the 2.748 cm. Hence here the observation 2.51 cm is said to be more accurate.

Now suppose you take 5 observations with both the instruments.

Instrument	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5
Vernier callipers	2.51 cm	2.53 cm	2.51 cm	2.47 cm	2.51 cm
Micrometer	2.748 cm	2.682 cm	2.749 cm	2.346 cm	2.747 cm

It is clear from the above table that most close value 2.555 cm is 2.53 cm, hence 2.53 cm is accurate, but observation 2.51 cm is observed for 3 times, hence 2.51 cm is precise. Here it is to be noted that accurate value is not precise and precise value is not accurate. In similar ways for Micrometer screw, observation 2.682 cm is close to 2.555 cm among other observations while, 2.748 ±0.001 cm is repeated 3 times hence it is more Precise observation.

Remember: Every accurate observation is not precise and every precise observation need not to be accurate.

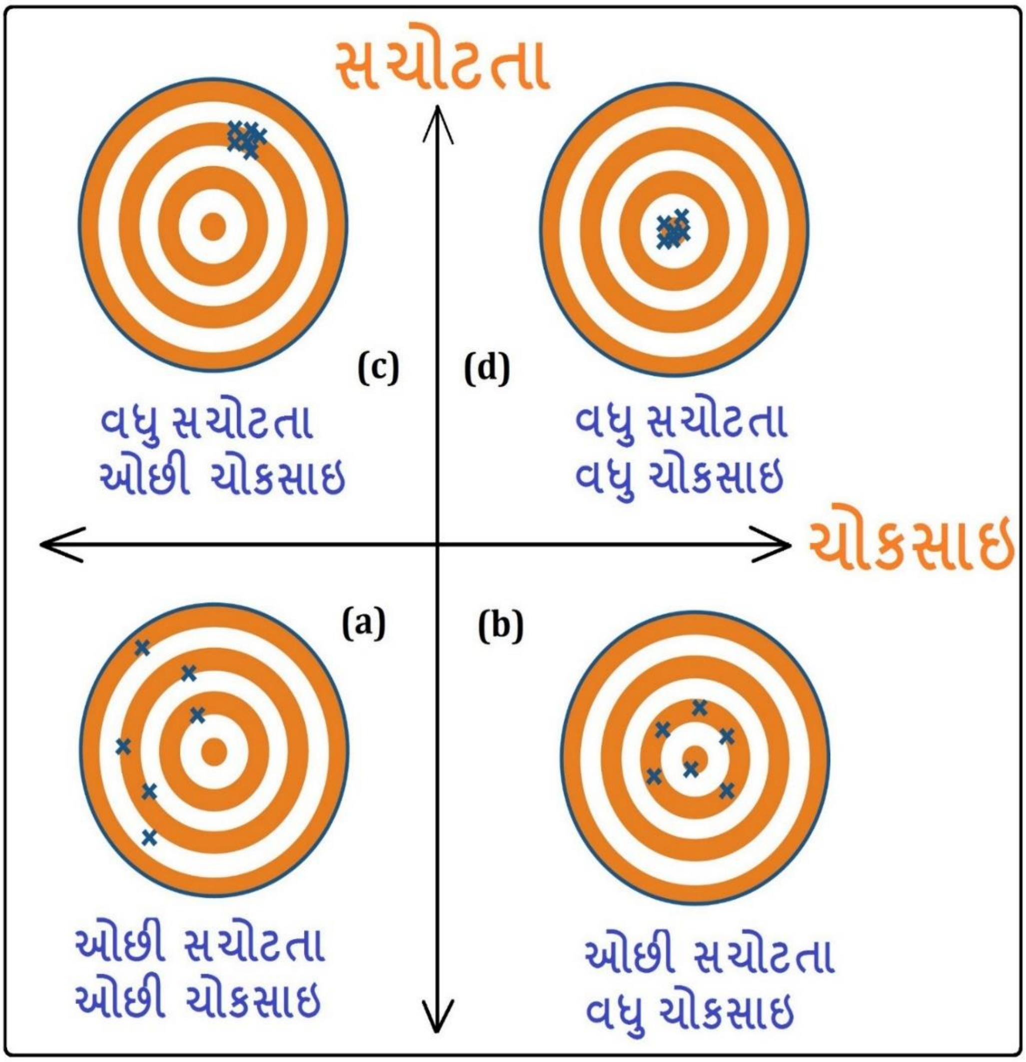
Least count of vernier calipers is 0.01 cm while the micrometer screw gauge has a least count of 0.001 cm. Measurements of micrometer screw gauges are more precise than those taken with vernier callipers.

Let us try to explain the difference between precision and accuracy with another example.

The diagram below shows the marks placed on the target in a rifle shooting competition.

Fig. (a): the marks are far from the center of the target (bull's eye) and also far from each other. So, it has less precision and less accuracy.

Fig (b): the marks are close to the center of the target (bull's eye) but far from



each other so, the accuracy is high but the accuracy is low as it is close to the true value.

Fig (c): the marks are far from the center of the target (bull's eye) but close together so, as they are far from the true value the precision is low but the accuracy is high.

Fig (d): the mark is closer to the center of the target (bull's eye) and also closer to each other, hence the mark is more precise and more accurate.

Error: Error is the difference between the true value of the physical quantity and the measured value. The error in measurement depends on the measurement method.

The error can be divided into two types.

1. **Systematic Errors:** Systematic errors arising during the measurement of physical quantity. Such errors cannot be both positive and negative. The reasons for the occurrence of these errors can be known and hence they can be removed to some extent. The reasons for the occurrence of systematic errors are as follows.

a) **Instrument error:** This type of error arises due to a defect in the instrument or a defect in the calibration of a scale of the instrument.

E.g. If the edge of the scale is broken or worn then, error arises regularly in the measurement.

An error occurs if the thermometer scale is not calibrated correctly.

b) **Error due to improper experimental method:** This type of error arises due to the improper method of conducting the experiment.

E.g. When body temperature is measured with the help of a thermometer, the thermometer kept under armpit shows lower temperature than body temperature.

c) **Personal Error:** This error arises due to the observation method of the person taking the observation, improper arrangement of the equipment, carelessness or inexperience in taking the observation.

E.g. The height of the fluid filled in the capillary, a true observation can be taken by keeping the eye in front of the meniscus of the fluid. Observations taken from above or below it does not give correct observations.

d) **Due to External Factors:** External factors such as temperature during the experiment, humidity, atmospheric pressure, air velocity etc. can also produce systematic error in the measurement.

Systematic errors in measurement can reduced by using high quality instruments, eliminating individual errors and improving the experiment method.

2. **Random Error:** This type of error arises in the observations taken due to irregular changes in the influencing factors during the experiment as well as unpredictable factors. This error can be positive or

negative. If any physical sign is measured repeatedly, not every observation will be the same, so an estimate of this type of error can be obtained by averaging several observations..

Estimation of Errors: When experimented with for the measurement of any physical sign, then since we do not know the true value of this physical zodiac in advance, the mean value of the observations of the experiment is taken as the true value.

a) **Absolute Error:** The difference between the true value (average value) and the experimental value (observation) of a physical quantity is called the absolute error of the observation.

Suppose a physical quantity is measured by observations $a_1, a_2, a_3, a_4, \ldots., a_n$

So, the average value $\bar{a}$,

$$\bar{a} = \frac{a_1 + a_2 + a_3 + a_4 +, \ldots., + a_n}{n}$$

$$\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

By definition the absolute error for each observation,

$$\Delta a_1 = \bar{a} - a_1$$

$$\Delta a_2 = \bar{a} - a_2$$

$$\Delta a_3 = \bar{a} - a_3$$

$$\vdots \qquad \vdots$$

$$\Delta a_n = \bar{a} - a_n$$

$\Delta a_1, \Delta a_2, \Delta a_3, \Delta a_4, \ldots.., \Delta a_n$ is called the absolute error of each observation. This error can be positive or negative.

b) **Mean Absolute Error:** The mean of the positive values of the absolute error of each observation is called the mean absolute error.

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + \cdots \ldots + |\Delta a_n|}{n}$$

$$\Delta\bar{a} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

Thus the measurement of a physical quantity can be expressed as follows.

$$a = \bar{a} \pm \Delta\bar{a}$$

It means the value of physical quantity will be between and $\bar{a} + \Delta\bar{a}$ & $\bar{a} - \Delta\bar{a}$.

c) **Relative or Fractional Error:** The ratio of the mean absolute error to the mean value is called the relative error of a physical quantity.

$$\delta a = \frac{\Delta\bar{a}}{\bar{a}}$$

d) **Percentage Error:** If the relative error is expressed as a percentage, it is called percentage error.

$$\delta a\ \% = \delta a \times 100\ \%$$
$$\delta a\ \% = \frac{\Delta\bar{a}}{\bar{a}} \times 100\ \%$$

Error Propagation: The mean value of any physical quantity is determined from a number of independently measured observations. For example, in the laboratory speed is measured as ratio of distance to time. In such situations a physical quantity measured associated with Errors that in error of physical quantity effect produced oing. In the end result of errors to estimate of errors Determination of diffusion kNeed to leavefallsis.

When any physical quantity is measured with instrument, there is always chance of uncertainty in measurement because each measuring instrument has limit to its least count. This leads to error in those measurements. Error propagation occurs when certain physical quantity is measured with error and this quantity is used to calculate other physical quantity.

For example, when we measure distance and time with some errors and when we use this distance and time to calculate the speed of the given object then, error in speed grow much more quickly than individual errors in distance and time.

Another example when you try to calculate the area of object by measuring its length and breadth with errors.

Suppose x and y are the average values of the quantities being measured, and in these average values Δx and Δy are the uncertainties.

Rule 1: Addition/Subtraction: Suppose the physical amount $f(x,y)$ A physical amount x If is obtained by addition or subtraction of and y ,

$$f(x,y) = x \pm y$$
$$\therefore \Delta f = \Delta x \pm \Delta y$$

For example, parallel connection of two resistances,

$$R_1 = 5.0 \pm 0.3\ \Omega, \qquad R_2 = 8.0 \pm 0.6\ \Omega$$
$$R_P = (R_1 + R_2) \pm (\Delta R_1 + \Delta R_2)$$
$$\therefore R_P = (5.0 + 8.0) \pm (0.3 + 0.6)\ \Omega$$
$$\therefore R_P = 13.0 \pm 0.9\ \Omega$$

Rule2: Multiplication/Subtraction: Assume a physical quantity $f(x,y)$. It is obtained by multiplication or division of x and y,

$$f(x,y) = x \times y \quad or \quad f(x,y) = \frac{x}{y}$$
$$\frac{\Delta f}{f} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

For example, $Speed = \frac{distance}{time}$

$$\frac{\Delta s}{s} = \frac{\Delta d}{d} + \frac{\Delta t}{t}$$

Rule – 3: Exponent/Exponential: Suppose a physical quantity $f(x, y)$ is obtained in the form x^m.

$$f(x, y) = x^m$$
$$\frac{\Delta f}{f} = |m|\frac{\Delta x}{x}$$

For example the equation of a parabola,

$$y = x^2$$
$$\frac{\Delta y}{y} = |2|\frac{\Delta x}{x}$$

Rule – 4: Constant: Suppose a physical quantity $f(x, y)$. A physical quantity is obtained in the form $c \cdot x$. where, c is constant.

$$f(x, y) = c \cdot x$$
$$\Delta f = c \cdot \Delta x$$

For example Equation of the line passing through the origin,

$$y = 5x$$
$$\Delta y = 5\Delta x$$

Significant Numbers: The digits indicating how many digits are reliable or significant in the measured value of any physical quantity are called significant digits. The greater the number of significant digits in the measurement of physical quantity., the greater the precision of the measurement.

As shown in the figure which container should be used to take 9.7 ml of liquid?

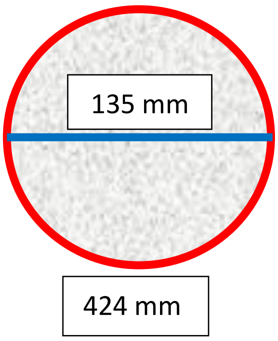
100 ml graduated cylinder and a graduated beaker of 100 ml size can measure at least 10 ml accurately, so if 9.7 ml of liquid is measured with these two containers, there is a possibility of uncertainty in measurement, while 10 ml A graduated cylinder capable of accurately measuring at least 1 ml. Hence it is less prone to errors than measurements made with two other instruments.

E.g. When the length of a piece of wood is measured with a simple tape measure, suppose value falls between 4.6 cm and 4.7 cm, then if we assume the length to be 4.65 cm, the number of significant digits in this observation is 3, of which 4 and 6 are accurate but the digit 5 is not known accurately, because the length of the simple scale is 0.1 cm. Thus, with scale a single digit after the decimal point of can be accurately measured. This leaves uncertainty in the measurement of digit 5.

Rules for Significant digits:

1) All Non-Zero (other than zero) digits are significant digits.
   E.g. The number of significant digits in is 4.$x = 2597$
2) All zeros between two non-zero digits are significant digits. E.g. in$x = 3.0046$ There are 5 significant digits.
3) All zeros to the left of the digits are not counted as significant digits. E.g.The number of significant digits in 0.0056 is 2. According to this rule the number of significant digits in 0177 is 3.
4) Zeros to the right of numbers without a decimal point do not count as significant digits. E.g.Number of significant digits in 15600 is 3. But when the digits have been measured as an exact measure of a physical quantity, all those digits (including zero) are considered significant digits. e.g., The number of significant digits in $R = 2000\ \Omega$ is 4, while the number of significant digits in $x = 2000$ is only 1.
5) If a number with a decimal sign has zeros on right hand side of non-zero digits. e.g., The number of significant digits in $x = 1.54000$ are 6.
6) Powers of 10 are not taken as significant digits. e.g., 1 and 6 are the only two significant digits in 1.6×10^{-19}.
7) Number of significant digits does not change by changing the units of measurement of physical quantity. e.g., Height of Mount Everest $8848\ m$. The number of significant digits in the measurement is 4. If this number is represented as $8.848\ km$ or $8.848 \times 10^5\ cm$, the number of significant digits remains 4 in both the cases.

The value of measurement	Number of significant digits	rule
53559	5	1
7004.3	5	2
0.0062	2	3
550 m	3	4
39100	3	4
2.40	3	5
3.6×10^6	2	6



Round Off: Suppose you want to check the numerical value of π. For this we need to calculate the ratio of the circumference to the diameter of a circle. The true value of π in ten digits is 3.141592654. To check this, draw a circle and measure its circumference and diameter values accurately. Suppose they get the values 424 mm and 135 mm. Here it can be said that unit digit 4 in 424 is uncertain, and unit digit 5 in 135 is uncertain.

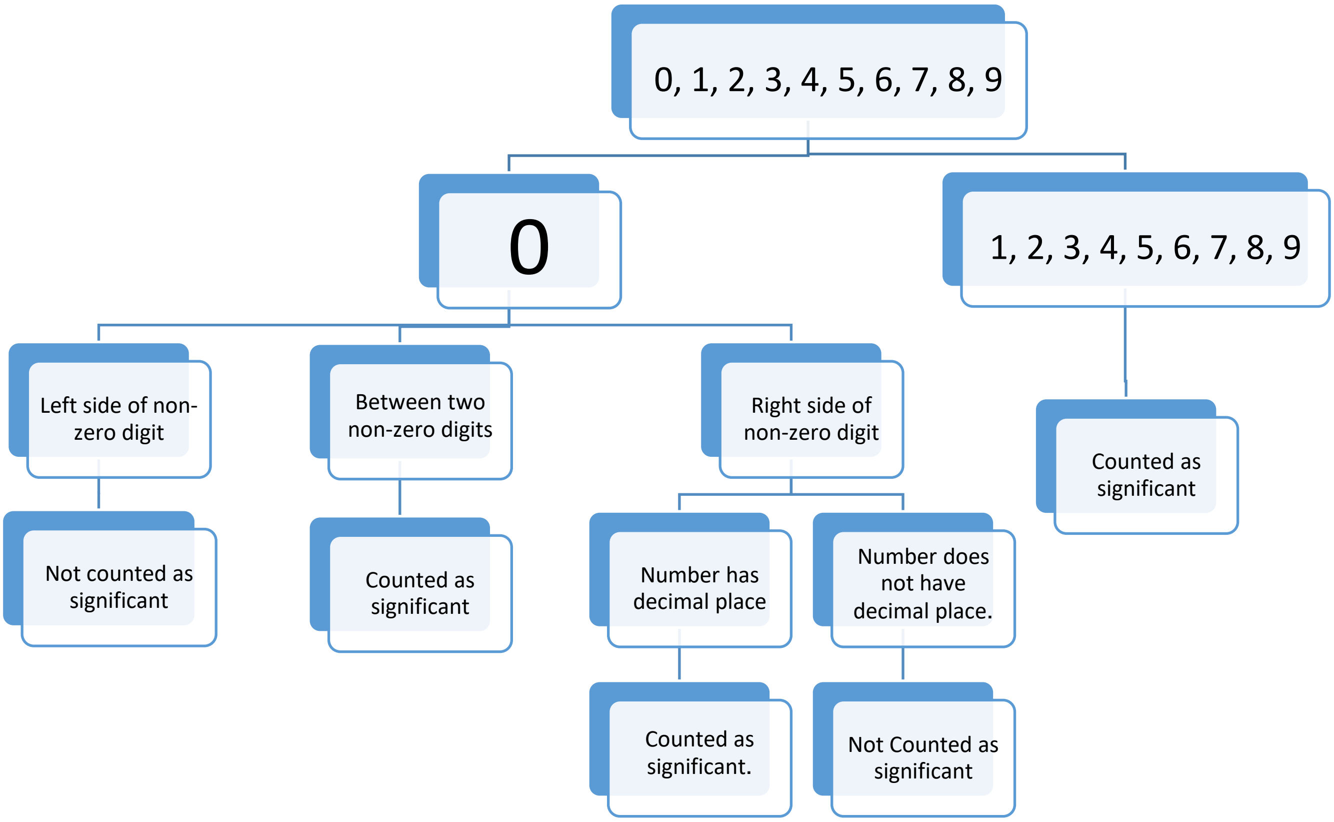
Now calculating the ratio of 424 and 135 using the calculator gives the value of 3.140740741 in ten digits. which does not match the true value of This is because the circumference and diameter values you measured have three significant digits. Hence the obtained value of 3.140740741 can be taken to only three significant digits which is 3.14.

$$\pi = \frac{Circumference}{Diameter} = \frac{424}{135} = 3.140740741$$

Rules:

1) If the last digit to be dropped in a given number is less than 5 then the preceding number does not change at all. e.g., $x = 6.24$ is obtained by rounding off to two significant digits.$x = 6.2$
2) If the last digit to be dropped in a given number is greater than 5, then the preceding digit is increased by 1. e.g., by rounding off to three significant digits$x = 6.468$ reduces to $x = 6.47$
3) It does not matter if the last digit to be dropped in a given number is 5 or 5 and zero to its right. e.g., Round off $x = 7.250$ to two significant digit gives $x = 7.25$ and round off to one significant digit it becomes $x = 7.2$.
4) If the last digit to be dropped in a given number is 5 and the preceding digit is odd then the number does not change. e.g., $x = 9.65$ is reduced to $x = 9.6$ to two significant digits. If the digit preceding 5 is odd then 1 is added to the preceding number. e.g., $x = 3.875$ rounding off to two significant digit reduces to $x = 3.88$.
5) "Number 0 (Zero) is an even number." Hence in a given number if the last digit to be dropped is 5 and if the preceding digit is 0 then according to rule (4) there is no change in that number. e.g., $x = 9.605$ by rounding off to two significant digits it becomes $x = 9.6$

The value of measurement	After round off, up to three digits	rule
7.364	7.36	1
8.437	8.44	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4
4.8405	4.84	5
8.730500	8.73	5



Numerical

1. 1 cm of the main scale of a vernier calipers is divided into 10 divisions. If the 10 divisions of the vernier scale coincides to the 9 divisions of the main scale, then find the least count of this instrument.

   Solution: 1 cm of the main scale is divided into 10 divisions hence the smallest value on the main scale will be,

   $$\frac{1\ cm}{10} = 0.1\ cm$$

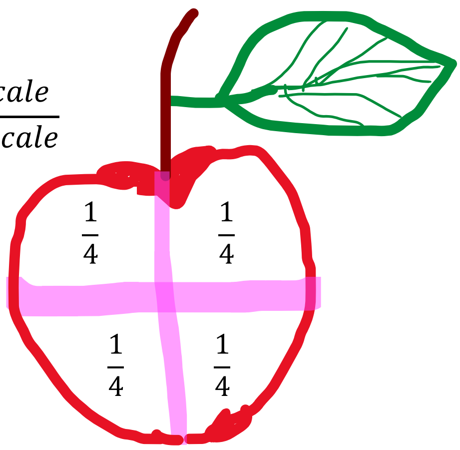
   Now, according to the formula of L.C. of the Vernier callipers,

   $$L.C. = \frac{Value\ of\ smallest\ division\ of\ Main\ scale}{Total\ number\ of\ division\ of\ vernier\ scale}$$

   $$\therefore L.C. = \frac{\frac{1\ cm}{10}}{10}$$

   $$\therefore L.C. = \frac{1\ cm}{100}$$

   $$L.C. = 0.01\ cm$$

   thus, L.C. of Vernier callipers is 0.01 cm.

2. 1 cm of the main scale of a Vernier callipers is divided into 20 divisions. If the 20 divisions of the vernier scale coincide to 19 divisions of the main scale, then find out least count of this instrument.

Solution: 1 cm of the main scale is divided into 20 divisions hence the smallest value on the main scale is $\frac{1}{20} cm$.

Now, according to the formula of L.C. of the Vernier callipers,

$$L.C. = \frac{Value\ of\ smallest\ division\ of\ Main\ scale}{Total\ number\ of\ division\ of\ vernier\ scale}$$

$$\therefore L.C. = \frac{\frac{1\ cm}{20}}{10}$$

$$\therefore L.C. = \frac{1\ cm}{200}$$

$$L.C. = 0.01\ cm$$

$$\therefore L.C. = \frac{1}{200} \times \frac{5}{5}\ cm = \frac{5}{1000} cm$$

$$\therefore L.C. = 0.005\ cm$$

thus, L.C. of vernier calipers 0.005 cm.

Important to remember:

- "1 on the main scale of vernier caliperscm is divided into 10 sections." Which usually means the main scale of vernier calipers is a simple scale, which has 10 divisions in 1 cm, hence the smallest value of measurement is $1\ mm = \ 0.1\ cm$ means $\frac{1}{10} cm$.
- "1 cm of the main scale of vernier calipers into 10 (or 20, 25, …) divisions is divided" means the smallest value of measurement measured by the main scale $\frac{1}{10} cm,\ \frac{1}{20} cm,\ \frac{1}{25} cm, \dots ..$

3. In an experiment to measure the diameter of a metal wire, the zero division of main scale of vernier calipers lies between 2.5 cm and 2.6 cm, and the 3rd division of the vernier scale coincides with any divisions of main scale. Find the cross-sectional area of this wire if the L.C. of the vernier callipers is 0.1 mm.

Solution: The values of main scale is taken as 2.5 cm as zeroth division lies between 2.5 cm and 2.6 cm, (we take the smaller values of measurement which is 2.5 cm)

$$M.S. = \ 2.5\ cm$$

$$L.C\ = 0.1\ mm = 0.01\ cm$$

$$D\ = M.S. + (V.S. \times L.C)$$

$$D\ = 2.5 + (3 \times 0.01) = 2.5 + 0.03 = 2.53\ cm$$

$$\therefore Diameter\ D = 2.53\ cm$$

$$Are\ of\ wire, A = \ \pi * r^2 = \pi * \left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} = \frac{3.14 * 2.53^2}{4}$$

$$A = 5.02470\ cm^2$$

It is necessary to round off the value of area to three digits, (because L.C. of the measuring instrument is L.C. = 0.01 cm which has two decimal point value)

$$\therefore A = 5.02\ cm^2$$

4. In an experiment to measure the length of a cube, the zeroth division of the main scale of the vernier callipers lies between 3.8 cm and 3.9 cm and 5th division of the vernier scale coincides with any division of main scale. Weight of Vernier Calipers 0.01 cm and the positive error is 3 kappa, find the volume of this cube.

Solution: Since the error here is positive, it has to be subtracted from the main scale observation.

$$\text{ધન ત્રુટિ} = \text{બંધ બેસતો કાપો} * \text{લ. મા. શ.}$$
$$\text{ધન ત્રુટિ} = 3 * 0.01 = 0.03\ \text{cm}$$
$$l' = M.S + (V.S.* L.C)$$
$$l' = 3.8 + (5 * 0.01)$$
$$l' = 3.85\ cm$$
$$l = l' - \text{ધન ત્રુટિ}$$
$$l = 3.85 - 0.03 = 3.82\ cm$$

Volume of a cube,

$$V = l^3$$
$$= (3.82)^3$$
$$V = 55.74\ cm^3$$

5. A main scale measuring the diameter of a sphere with vernier calipersfalls between 4.3 cm and 4.4 cm, and falls within 2 divisions of its vernier scale. L.M.Sh of this tool. 0.01 cm and the negative error is 3 kappa, find the volume and surface area of this sphere.

Solution: $$D' = M.S. + (V.S. \times L.C)$$

$$D' = 4.3 + (3 \times 0.01) = 4.3 + 0.03$$

$$D' = 4.33\ cm$$

$$\text{ઋણ ત્રુટિ} = \text{બંધ બેસતો કાપો} * \text{લ. મા. શ.}$$
$$\text{ઋણ ત્રુટિ} = 3 * 0.01 = 0.03\ \text{cm}$$
$$D' = M.S + (V.S.* L.C)$$
$$D = D' + \text{ઋણ ત્રુટિ}$$
$$D = 4.33 + 0.03 = 4.36\ cm$$

The wealth of the sphere

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3} \times \pi \times \left(\frac{D}{2}\right)^3 = \frac{4}{3} \times \frac{\pi D^3}{8} = \frac{\pi D^3}{6} = \frac{3.14 \times 4.36^3}{6} = 43.37483\ cm^3$$
$$V = 43.38\ cm^3$$

Outer surface area of sphere:

$$A = 4\pi r^2$$

$$A = 4\pi \left(\frac{D}{2}\right)^2 = \pi D^2 = 3.14 \times 4.36^2 = 59.690144\ cm^2$$

$$A = 59.69\ cm^2$$

thus, the volume and surface area of the sphere are found.$V = 43.38\ cm^3$ $A = 59.69\ cm^2$

6. to the main scale of a micrometer screw gaugeis marked in mm and has 100 divisions on its circular scale, then the weight of this instrument is Search in micrometers.

   Solution: L of micrometer screw gauge.

$$L.C. = \frac{Value\ of\ smallest\ division\ on\ main\ scale\ (Pitch)}{Total\ number\ of\ divisions\ on\ circular\ scale}$$

   *(Circular scale is also known as Head scale and conventionally also as Vernier scale)

$$L.C. = \text{લ. મા. શ.} = \frac{\text{મુખ્ય સ્કેલ પરનું નાનામાં નાનું માપ (પીચ)}}{\text{વર્નિયર સ્કેલ પરના કુલ કાપા(વિભાગ) ની સંખ્યા}}$$

$$L.C. = \frac{1}{100} mm = 0.01\ mm$$

$$L.C. = 0.01\ mm$$

$$= 10^{-2}\ mm$$

$$= 10^{-2} \times 10^{-1} cm$$

$$= 10^{-2} \times 10^{-1} \times 10^{-2}\ m$$

$$= 10^{-5}\ m$$

$$= 10 \times 10^{-6}\ m$$

$$L.C. = 10\ \mu m$$

7. A micrometer screw pitchis 1 mm. Its L.M.Sh. 0.02 mm, find the number of cuts on its head scale.

   Solution:પીચ $= 1$ mm અને લ. મા. શ. $= 0.02\ mm$

$$L.C. = \text{લ. મા. શ.} = \frac{\text{પીચ}}{\text{વર્નિયર(હેડ) સ્કેલ પરના કુલ કાપા(વિભાગ) ની સંખ્યા}}$$

$$\therefore \text{વર્નિયર(હેડ) સ્કેલ પરના કુલ કાપા(વિભાગ) ની સંખ્યા} = \frac{\text{પીચ}}{\text{લ. મા. શ}}$$

$$\therefore \text{હેડ સ્કેલ પરના કુલ કાપા ની સંખ્યા} = \frac{1\ mm}{0.02\ mm} = 50$$

   Thus the number of notches on the micrometer screw head scale is 50.

8. A micrometer screw L.M.sh.0.01 mm and the number of cuts on its head scale is 200. is 1 mm. Find out its pitch.

   Solution:હેડ સ્કેલ પરના કુલ કાપાની સંખ્યા $= 1$ mm

$$\text{લ. મા. શ.} = 0.02\ mm$$

$$\text{લ. મા. શ.} = \frac{\text{પીચ}}{\text{હેડ સ્કેલ પરના કુલ કાપાની સંખ્યા}}$$

$$\text{પીચ} = \text{લ. મા. શ.} \times \text{હેડ સ્કેલ પરના કુલ કાપાની સંખ્યા}$$

$$\text{પીચ} = 0.01 \times 200\ mm$$

$$\text{પીચ} = 2\ \text{mm}$$

thus, the pitch of a given micrometer is found to be 1 mm.

9. Diameter of a sphereIts main scale, measured with a micrometer, falls between 4 mm and 5 mm, and falls on the 57th division of its circular scale. L.M.Sh of this tool. 0.01 mm and the negative error is 3 kappa, find the volume and surface area of this sphere.

Solution: $$D' = M.S. + (V.S. \times L.C)$$

$$D' = 4 + (57 \times 0.01) = 4 + 0.57$$

$$D' = 4.57\ mm$$

$$\text{ઋણ ત્રુટિ} = \text{બંધ બેસતો કાપો} * \text{લ. મા. શ.}$$

$$\text{ઋણ ત્રુટિ} = 3 * 0.01 = 0.03\ \text{mm}$$

$$D' = M.S + (V.S. * L.C)$$

$$D = D' + \text{ઋણ ત્રુટિ}$$

$$D = 4.57 + 0.03 = 4.60\ mm$$

The wealth of the sphere

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3} \times \pi \times \left(\frac{D}{2}\right)^3 = \frac{4}{3} \times \frac{\pi D^3}{8} = \frac{\pi D^3}{6} = \frac{3.14 \times 4.57^3}{6} = 50.939173\ mm^3$$

$$V = 50.94\ mm^3$$

Outer surface area of sphere:

$$A = 4\pi r^2$$

$$A = 4\pi \left(\frac{D}{2}\right)^2 = \pi D^2 = 3.14 \times 4.60^2 = 66.4424\ mm^2$$

$$A = 66.44\ mm^2$$

10. Following are the observations made in the experiment of measuring the refractive index of water. If found, the absolute error for these observationsη: $1.28, 1.30, 1.32, 1.34, 1.36, 1.38$, find the mean absolute error, relative error and percentage error.

Solution: Assume that if the mean value of the refractive index is$\bar{n}$,

$$\bar{n} = \frac{n_1 + n_2 + n_3 + n_4 + n_5 + n_6}{6}$$

$$\bar{n} = \frac{1.28 + 1.30 + 1.32 + 1.34 + 1.36 + 1.38}{6}$$

$$\bar{n} = \frac{7.98}{6}$$

$$\bar{n} = 1.33$$

Absolute error for each observation,

$$\Delta n_1 = \bar{n} - n_1 = 1.33 - 1.28 = 0.05$$

$$\Delta n_2 = \bar{n} - n_2 = 1.33 - 1.30 = 0.03$$

$$\Delta n_3 = \bar{n} - n_3 = 1.33 - 1.32 = 0.01$$

$$\Delta n_4 = \bar{n} - n_4 = 1.33 - 1.34 = -0.01$$

$$\Delta n_5 = \bar{n} - n_5 = 1.33 - 1.36 = -0.03$$

$$\Delta n_6 = \bar{n} - n_6 = 1.33 - 1.38 = -0.05$$

Mean absolute error:

$$\Delta\bar{n} = \frac{|\Delta n_1| + |\Delta n_2| + |\Delta n_3| + |\Delta n_4| + |\Delta n_5| + |\Delta a_6|}{6}$$

$$\Delta\bar{n} = \frac{|0.05| + |0.03| + |0.01| + |-0.01| + |-0.03| + |-0.05|}{6}$$

$$\Delta\bar{n} = \frac{0.05 + 0.03 + 0.01 + 0.01 + 0.03 + 0.05}{6}$$

$$\Delta\bar{n} = \frac{0.18}{6}$$

$$\Delta\bar{n} = 0.03$$

સાપેક્ષ ત્રુટિ $\delta a = \frac{\Delta\bar{a}}{\bar{a}}$

સાપેક્ષ ત્રુટિ $\delta a = \frac{0.03}{1.33} = 0.023$

પ્રતિશત ત્રુટિ $= \delta a \times 100\ \%$

પ્રતિશત ત્રુટિ $= 0.023 \times 100$

પ્રતિશત ત્રુટિ $= 2.3\ \%$

11. Following are the observations made in an experiment of measuring the length of a metal wire with vernier calipers.If found, find the absolute error, mean absolute error, relative error and percentage error for these observations.$l = 2.50\ cm, 2.54\ cm, 2.58\ cm, 2.62, cm, 2.66\ cm$

Solution: Assume that if the average value of the length$\bar{l}$,

$$\bar{l} = \frac{l_1 + l_2 + \ l_3 + \ l_4 + l_5}{5}$$

$$\bar{l} = \frac{2.50 + 2.54 + 2.58 + 2.62 + 2.66}{5}$$

$$\bar{l} = \frac{12.9}{5}$$

$$\bar{l} = 2.58\ cm$$

Absolute error for each observation,

$$\Delta l_1 = \bar{l} - l_1 = 2.58 - 2.50 = 0.08\ cm$$

$$\Delta l_2 = \bar{l} - l_2 = 2.58 - 2.54 = 0.04\ cm$$

$$\Delta l_3 = \bar{l} - l_3 = 2.58 - 2.58 = 0.00\ cm$$

$$\Delta l_4 = \bar{l} - l_4 = 2.58 - 2.62 = -0.04\ cm$$

$$\Delta l_5 = \bar{l} - l_5 = 2.58 - 2.66 = -0.08\ cm$$

Mean absolute error:

$$\Delta\bar{l} = \frac{|\Delta l_1| + |\Delta l_2| + |\Delta l_3| + |\Delta l_4| + |\Delta l_5|}{5}$$

$$\Delta\bar{n} = \frac{|0.08| + |0.04| + |0.00| + |-0.04| + |-0.08|}{5}$$

$$\Delta\bar{l} = \frac{0.08 + 0.04 + 0.00 + 0.04 + 0.08}{5}$$

$$\Delta\bar{l} = \frac{0.24}{5}$$

$$\Delta\bar{l} = 0.048\ cm$$

$$\text{સાપેક્ષ ત્રુટિ}\ \delta l = \frac{\Delta\bar{l}}{\bar{l}}$$

$$\text{સાપેક્ષ ત્રુટિ}\ \delta l = \frac{0.048}{2.58} = 0.019$$

$$\text{પ્રતિશત ત્રુટિ} = \delta l \times 100\ \%$$

$$\text{પ્રતિશત ત્રુટિ} = 0.019 \times 100$$

$$\text{પ્રતિશત ત્રુટિ} = 1.9\ \%$$

12. Following are the observations made in an experiment to measure the period of a simple pendulum.If found, find the absolute error, mean absolute error, relative error and percentage error for these observations.T: $1.96\ s, 1.98\ s, 2.00\ s, 2.02\ s, 2.04\ s$

Solution: Assume that if the average value of the period is$\bar{T}$,

$$\bar{T} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$

$$\bar{T} = \frac{1.96 + 1.98 + 2.00 + 2.02 + 2.04}{5}$$

$$\bar{T} = \frac{10.00}{5}$$

$$\bar{T} = 2.00\ s$$

Absolute error for each observation,

$$\Delta T_1 = \bar{T} - T_1 = 2.00 - 1.96 = 0.04\ s$$

$$\Delta T_2 = \bar{T} - T_2 = 2.00 - 1.98 = 0.02\ s$$

$$\Delta T_3 = \bar{T} - T_3 = 2.00 - 2.00 = 0.00\ s$$

$$\Delta T_4 = \bar{T} - T_4 = 2.00 - 2.02 = -0.02\ s$$

$$\Delta T_5 = \bar{T} - T_5 = 2.00 - 2.04 = -0.04\ s$$

Mean absolute error:

$$\Delta\bar{T} = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5}$$

$$\Delta\bar{T} = \frac{|0.04| + |0.02| + |0.00| + |-0.02| + |-0.04|}{5}$$

$$\Delta\bar{T} = \frac{0.04 + 0.02 + 0.00 + 0.02 + 0.04}{5}$$

$$\Delta\bar{T} = \frac{0.12}{5}$$

$$\Delta\bar{T} = 0.024\ s$$

$$\text{સાપેક્ષ ત્રુટિ}\ \ \delta T = \frac{\Delta\bar{T}}{\bar{T}}$$

$$\text{સાપેક્ષ ત્રુટિ}\ \ \delta T = \frac{0.024}{2.00} = 0.012$$

$$\text{પ્રતિશત ત્રુટિ} = \delta T \times 100\ \%$$

$$\text{પ્રતિશત ત્રુટિ} = 0.012 \times 100$$

$$\text{પ્રતિશત ત્રુટિ} = 1.2\ \%$$