CHAPTER – 2
FORCE & CIRCULAR MOTION

FORCE:

Force in everyday terms means push or pull. A better definition of force is the interaction between two physical objects or an object and its environment. For such reasons that the force exerted by one object on another is considered. When you push a bench in a classroom, you exert a force on the bench. Under the effect of such a force the bench moves. Thus, applying a force changes the state of the object.

DISTANCE AND DISPLACEMENT: Let us understand the difference between these two physical terms with an example. Suppose an elderly person is doing a “morning walk” in the early morning on a semi-circular track as shown in the figure. The radius of this track is 7 m. So when this person reaches from point A to point B, the total distance covered by him will be equal to half the circumference of this semicircle

Semi-circumference of a circle

$$S = \frac{2\pi r}{2} = \pi r = \frac{22}{7} * 7 = 22\ m$$

Diameter of circle D=2r=2*7=14 m.

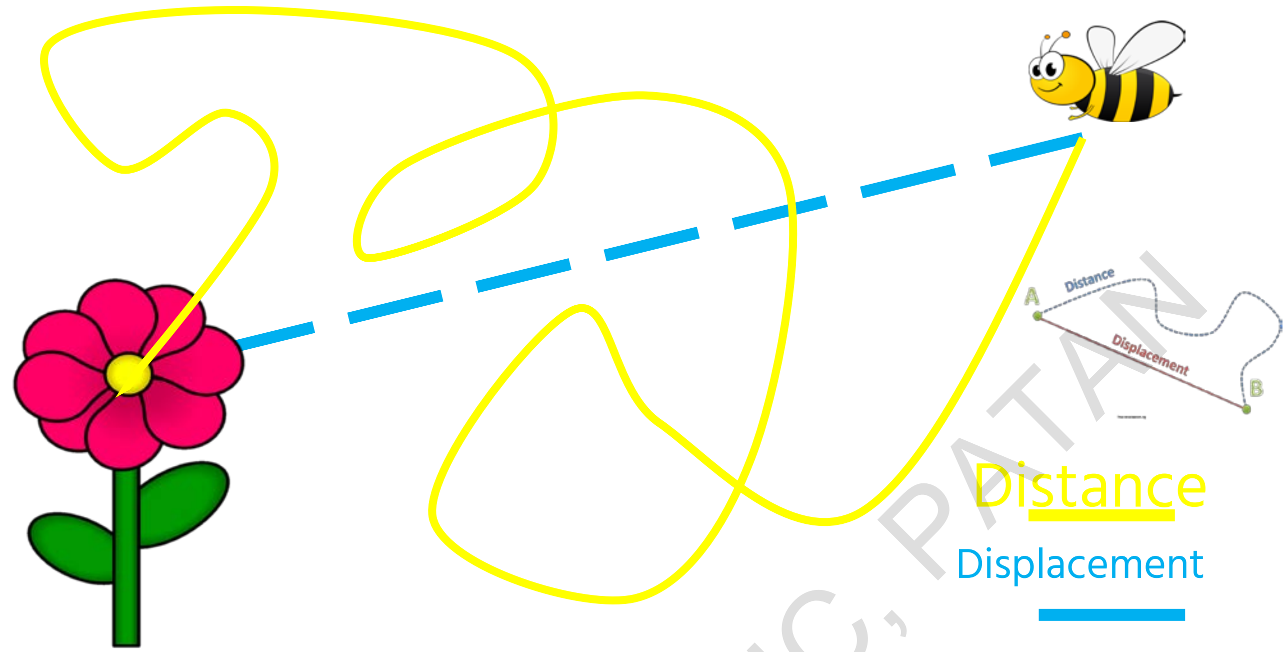
Thus, the total distance covered by this person will be 22 m and displacement will be 14 m.

DISPLACEMENT: A straight line connecting two points in the path of motion is known as displacement of the object.

DISTANCE: The total length of the entire path of motion is called the total distance traveled by the object.

REMEMBER:

1. Distance is scalar, while Displacement is vector.
2. Distance is always positive, while displacement can be positive, negative or even zero.



NEWTON'S FIRST LAW OF MOTION:

The first law gives an idea of how the forces acting on a physical object affect the motion of that object. When the force acting on an object is zero, we conclude that the object is stationary. But how will the object move when the surface force acting on the moving object is zero?

"A stationary object remains at rest and a moving object maintains its momentum as long as the net external force acting on the object is zero."

Example: When a stationary bus suddenly starts moving, the passenger in the bus leans backwards, because the passenger in the stationary bus is also stationary. When the bus suddenly accelerates, the passenger's legs move with the bus, but his upper body from the waist up tries to remain stationary according to Newton's first law (law of inertia). Hence his upper body leans backwards as the leg part moves. Similarly, when a moving bus is suddenly braked, the passenger's whole body is in motion, but as soon as the brakes are applied, his legs become fixed with the bus, while his upper body, being in motion, tries to remain in motion as per Newton's law, hence his upper body part leans forward. Thus, Newton's first law of motion defines force.

Inertia: You may have noticed that leaves and fruits on the tree fall down when the tree is shaken by the trunk. Or when an old used carpet is hung up and shaken with a stick, the dust particles are blown away. Do you know why this happens?

According to Newton's first law of motion, *"Until an unbalanced external force is applied to the object, a stationary object remains at rest and a moving object remains in constant motion maintaining its constant speed and direction."*

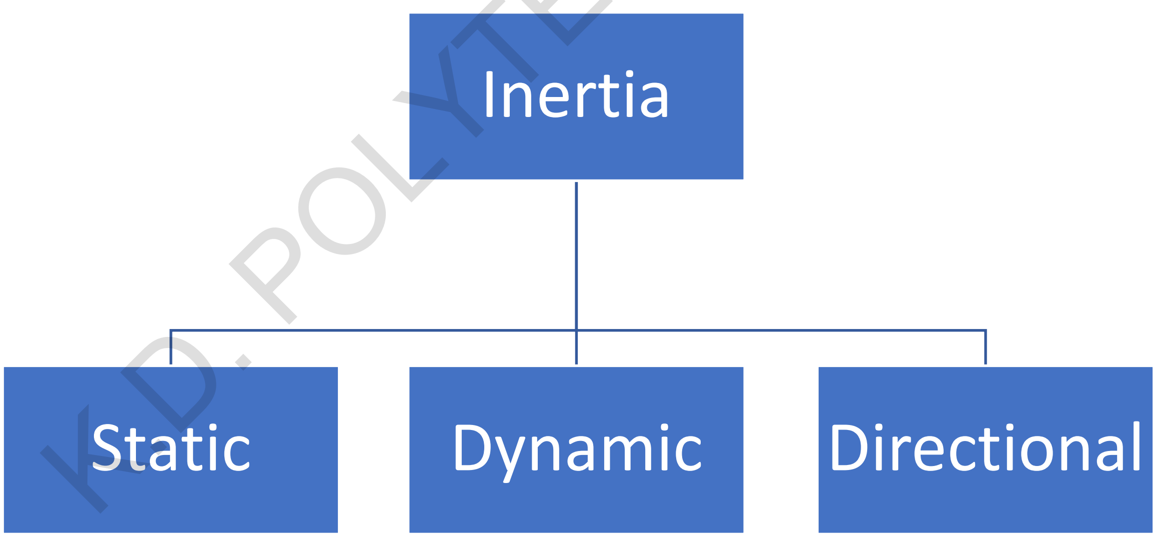
Simply put, "a physical object keeps doing what it used to do."

In fact, opposing one's state of motion (static or moving) is a fundamental characteristic of a physical object. The property of physical objects to oppose their own motion is called Inertia.

Mass as a Measure of the Amount of Inertia: All objects oppose their state of motion. Thus, all physical objects have a common tendency – **Inertia**. But some objects oppose their motion more or less than others. The characteristic of an object to resist its own motion varies with the mass of the object.

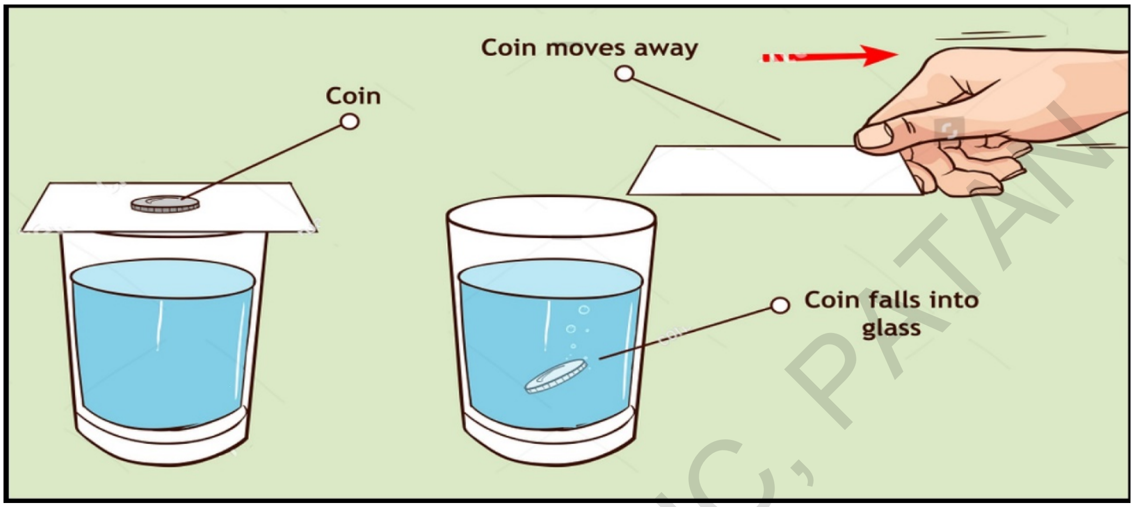
1. A mass is such a physical quantity that depends entirely on the inertia of that material.
2. The greater the inertia of the object, the greater its mass.
3. As mass increases, so does the object's tendency to oppose its own motion.

Example: A deer fleeing for its life has less mass than a cheetah chasing its prey, so it has less inertia (dynamic and directional) than a cheetah. Hence the deer can change direction quickly while running. Since the cheetah's high inertia prevents it from changing direction quickly, it is often unable to catch up to a rapidly changing deer (despite being the fastest predator on earth).



1. **<u>Static Inertia:</u>** The inability of an object in a stationary position to change its position is called the inertia of an object.

   <u>Example:</u> A coin on a card placed on a glass has positional inertia. If card is suddenly moved, the card is set in motion but, due to static (positional) inertia, the coin tries to stay still and eventually falls into the glass.



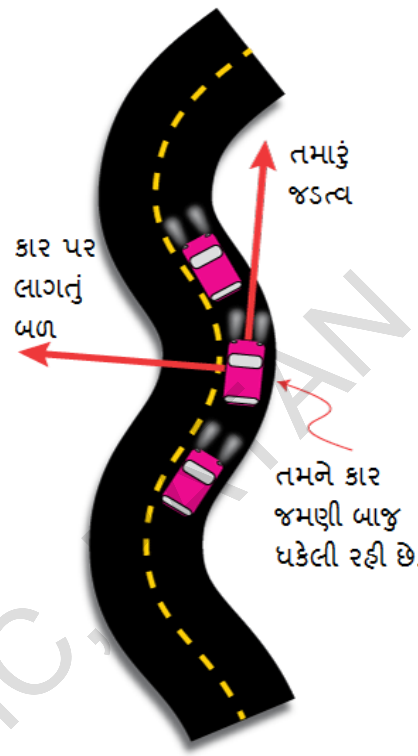
2. **<u>Dynamic Inertia:</u>** The inability of an object in motion to change its state of motion is called dynamic inertia of an object.

   <u>Example:</u> If a racing bike rider hits an obstacle safety-wall (a wall made of waste tires), the rider's kinetic inertia causes him to bounce off the bike and fling himself forward.

3. **<u>Directional Inertia:</u>** The inability of an object to change its direction of motion rather than moving along a fixed path is called directional inertia of an object.

Example:

- You have sometimes felt that you are pushed to the right when the car makes a sharp left turn. What is ultimately the virtual force pushing you to the right and where does it originate?
- When your car is moving in a straight line, you are also moving in a straight direction (the direction of the headlights) due to the directional inertia of your body.
- But when the car makes a sharp left turn your body tries to keep moving in a straight line due to directional inertia. So eventually you get pushed to the right.



Momentum:

The product of the velocity of a physical object and its mass is called the momentum of that object.

$$P = m \cdot v$$

Let us try to understand velocity with an example.

Case 1: Suppose two boys Motu $(m = 80\ kg)$ and Patalu $(m = 40\ kg)$ are running with same velocity $(v = 15\ m\ /\ s)$.

- Although their velocities are the same, their velocities will be different due to different masses.
- As the mass of the Motu is greater, its momentum will also be greater relative to Patalu.
- The Motu will require much more force than Patalu to stop both running runners. Thus, Motu running at a velocity of 15 m/s has a value of 1200 $\frac{kg \cdot m}{s^2}$ to 0 when it comes to rest $(v = 0\ m\ /\ s)$.
- Thus, the momentum difference gets larger in case of Motu which gives the value of the force required to stop Motu.

Let us try to understand velocity with another example.

Case 2: Suppose two boys Motu ($m = 80\ kg$) and Patalu ($m = 40\ kg$) are running with **same momentum**.

- Although their momentum is the same, their velocities will be different due to different masses.
- Since Motu has more mass, its velocity will be less relative to Patalu.
- To stop both runners, the same force would be required for both. Thus, the value of momentum is 800 $\frac{kg \cdot m}{s^2}$ to 0 when a mass moving with the same velocity comes to rest ($v = 0\ m\ /\ s$) .
- Thus, the momentum difference is equal in both cases. Which shows that the value of the force required to stabilize the objects moving at the same momentum is also the same.

Impact of a force: The product of the force $\vec{F}$ acting on an object and the period over which it is acting is called the impact of a force.

From Newton's second law,

$$\vec{F} = \frac{\Delta\vec{P}}{\Delta t}$$

$$Impulse = \vec{F} \cdot \Delta t = \Delta\vec{P} = m\Delta\vec{v} = m(\vec{v}_2 - \vec{v}_1)$$

Definition: *The product of a force and the value of the contact time of that force is called the impulse of the force and is equal to the product of the change in mass and velocity of the object.* When studying the collision of two or more objects, the average value of impulse of force is found. Changes in mass and velocity of objects during collisions are easily detectable, but forces are not. Hence if the contact time Δt can be obtained and from that value of force can be found.

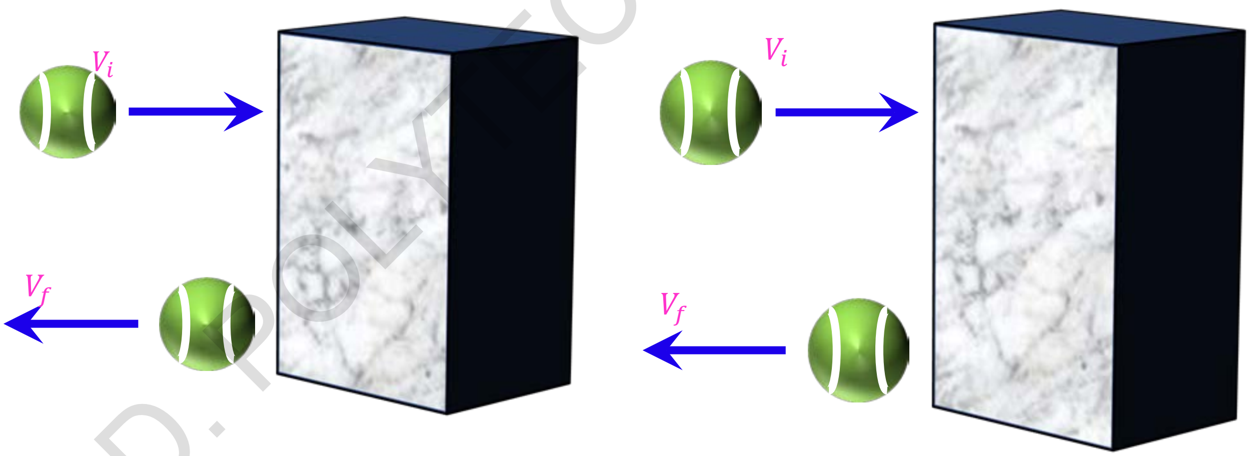
Example: When a tennis ball hits the racket during its motion, it stays connected for a few moments. This micro period is called contact time (Δt). The force with which this tennis ball is being hit gives the force value (F).

Thus, according to the equation $\vec{F} \cdot \Delta t = \Delta\vec{P}$ both the value of the tennis ball's velocity and the direction of the velocity change with the impact of the force $\left(\vec{F} \cdot \Delta t\right)$

• Thus, the multiplication of the tennis ball's velocity and its change in direction and the tennis ball's mass gives the value of its change in momentum, which is equal to the impulse of force acting on it.

Case -

Case -



ભૌતિક પરિબળો	Case - A	Case - B
સૌથી વધુ વેગમાં ફેરફાર	Y	N
સૌથી વધુ પ્રવેગમાં ફેરફાર	Y	N
સૌથી વધુ વેગમાનમાં ફેરફાર	Y	N
સૌથી વધુ બળના આઘાતમાં ફેરફાર	Y	N

Law of Conservation of Momentum:

The concept of momentum is useful when two or more objects are interacting in the physical world.

Two interacting particles exert a force on each other. According to Newton's third law of motion these forces are equal in value and opposite to each other. According to the second law, the magnitudes of their resultant forces are equal and opposite to each other, so the magnitudes of their velocity differences are equal and opposite to each other.

Statement: *"The total momentum of an isolated system remains constant."*

A bullet fired from a rifle propels the rifle back as it moves forward. If the force exerted by the rifle on the bullet is $\vec{F}$ then the force exerted by the bullet on the rifle is $-\vec{F}$. Both the rifle and the bullet are stationary before the bullet is fired. Hence taking their initial velocities $\vec{P}_r$ and $\vec{P}_b$,

$$\vec{P}_r + \vec{P}_b = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Taking the momentum of the rifle and bullet after release from the rifle as $\vec{P}_r{}'$ and $\vec{P}_b{}'$ respectively Now from Newton's second law of motion, the change in velocity of the bullet,

$$\vec{P}_b{}' - \vec{P}_b = \vec{F} \cdot \Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The change in momentum of the rifle,

$$\vec{P}_r{}' - \vec{P}_r = -\vec{F} \cdot \Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Summing equations (2) and (3),

$$\vec{P}_b{}' - \vec{P}_b + \vec{P}_r{}' - \vec{P}_r = \vec{F} \cdot \Delta t - \vec{F} \cdot \Delta t$$

$$\vec{P}_b{}' - \vec{P}_b + \vec{P}_r{}' - \vec{P}_r = 0$$

$$\vec{P}_b{}' + \vec{P}_r{}' = \vec{P}_r + \vec{P}_b$$

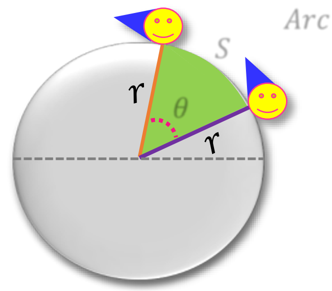
$$Final\ momentum\ of\ (bullet + rifle) = Initial\ momentum\ of\ (bullet + rifle).$$

Here, since no external force acts on (rifle + bullet), the system consisting of (rifle + bullet) can be taken as "isolated system". Here the fact is taken into account that the resultant effect of the internal forces acting on the system consisting of (rifle + bullet) is zero. This law is fundamental and universal and can be applied to star and planetary systems as well.

CIRCULAR MOTION

If a force is applied to an object having a fixed axis, it will move in a circular motion about its axis. Motion of a stone tied by a string, motion of wheels in a machine, etc. are examples of constant circular motion.

Suppose an object is moving in a circular path of radius r. Its angular position relative to the reference line at time t_1 is θ_i while its position at time t_2 is θ_f.



Angular Displacement: *The angle between the final and initial position for a given object relative to a fixed point in circular motion is called the angular displacement of that object at that time.*

$$\theta = \theta_f - \theta_i \dots\dots\dots(1)$$

Here the object travels a distance of arc S in a time interval $\Delta t = t_2 - t_1$. Hence the angle made by the object with the reference line in time Δt,

$$\theta = \frac{Arc}{Radius} = \frac{S}{r} \quad \dots\dots\dots(2)$$

$$\therefore S = r \cdot \theta \quad \dots\dots\dots(3)$$

The distance traveled by an object when it starts from a fixed point and returns to the same point where it started the journey is,

$$S = 2\pi r$$

Meanwhile during this complete journey, the angle subtended by the object,

$$\theta = 360°$$

Putting both the above values in eq. (3),

$$\therefore 2\pi r = r(360°)$$

$$\therefore 2\pi = 360°$$

$$\therefore \pi = 180°$$

Angular displacement (angle) in circular motion is measured in Radian (radian) unit.

The table given here is for information only.

Radian	Degree
0	0
$\pi/6$	30°
$\pi/4$	45°
$\pi/3$	60°
$\pi/2$	90°
π	180°
$3\pi/2$	270°
2π	360°

Conversion of degrees (D) and radians (R) to each other

$$D \xrightarrow{\times \frac{\pi}{180°}} R \qquad R \xrightarrow{\times \frac{180°}{\pi}} D$$

Angular Velocity: If the object covers an angular displacement of θ in time t, then the angular velocity of the object,

$$\text{Angular velocity } \omega = \frac{\text{Angular Displacement}}{\text{time}} = \frac{\theta}{t}$$

Unit of angular velocity : ${}^{rad}/_{s}$

Relationship between Linear Velocity and Angular Velocity:

According to the definition of linear velocity,

$$velocity = \frac{Displacement}{time}$$

Here the object travels a distance of arc $\boldsymbol{S}$ in a time interval to $\Delta t = t_2 - t_1$ and subtends an angle equal to Δθ.

$$\therefore \Delta S = r \cdot \Delta\theta$$

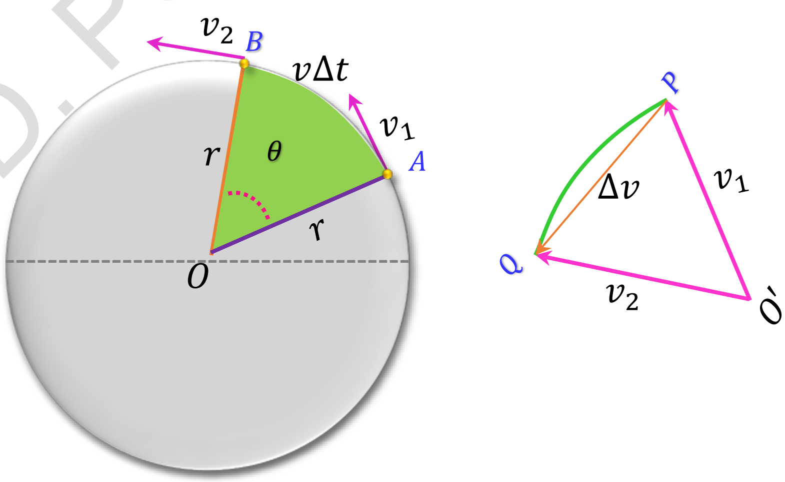
$$\therefore v = \frac{r \cdot \Delta\theta}{\Delta t}$$

$$\therefore v = r\frac{\Delta\theta}{\Delta t}$$

$$\therefore v = r\omega$$

Centripetal Acceleration:

The direction of the linear velocity of an object moving in a circular motion is constantly changing hence, circular motion can be said to be a uniformly accelerated motion.



As shown in the figure, an object is moving in a circular path of radius $\boldsymbol{r}$ with constant velocity. A tangent drawn to a circle shows the direction of the object's linear velocity at that point. Point A has velocity $\boldsymbol{v_1}$ and point B has velocity $\boldsymbol{v_2}$.

An object covers a distance of arc $\widehat{\boldsymbol{AB}}$ with velocity $\boldsymbol{v}$ in time $\Delta\boldsymbol{t}$. From the definition of velocity,

$$velocity = \frac{Displacement}{time} \rightarrow v = \frac{\widehat{AB}}{\Delta t}$$

$$\Rightarrow \widehat{AB} = v \cdot \Delta t$$

Here $\Delta\, OAB \cong \Delta\, O'PQ,$

$$\frac{AB}{OA} = \frac{PQ}{O'P}$$

$$\therefore \frac{v \cdot \Delta t}{r} = \frac{\Delta v}{v_1}$$

For circular motion only the direction of the object's velocity changes compared to motion with constant velocity, hence

$$\therefore \frac{v \cdot \Delta t}{r} = \frac{\Delta v}{v}$$

$$\therefore \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Now from the definition of acceleration,

$$Acceleration\ a = \frac{\text{વેગ}}{\text{સમય}}$$

$$\therefore a_c = \frac{v^2}{r}$$

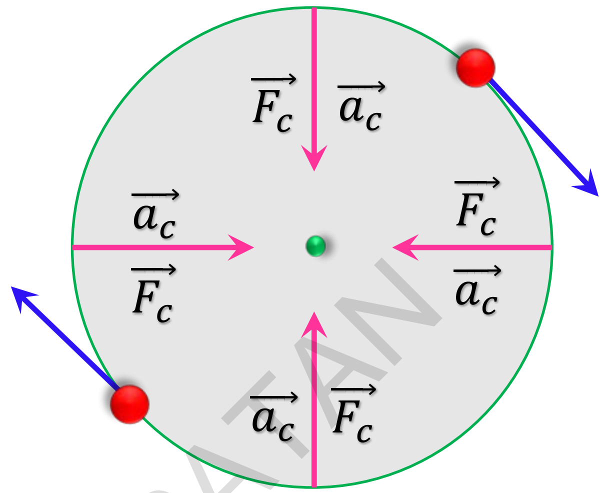
Linear velocity of an object

$$v = r\omega$$

$$\therefore a_c = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r}$$

$$\therefore a_c = r\omega^2$$

Centripetal Acceleration Unit: m/s^2



Centripetal force: Centripetal force for an object in circular motion According to Newton's law of motion,

$$F = ma$$

$$\therefore F_C = \frac{mv^2}{r} = mr\omega^2$$

Centripetal force is responsible for the regular constant acceleration of an object. Velocity and distance of an object in circular motion are always constant hence the value of centripetal force also remains constant. Since only the direction of the velocity changes continuously, the direction of the centripetal force also changes continuously but the direction of the centripetal force is always towards the center.

Centripetal Force unit: $\frac{kg \cdot m}{s^2} = N$

Examples:

1. Marry-go-round
2. Motion of the Earth
3. Traction speed
4. Washing machine speed

Banking of the road:

A vehicle moving on a curved road is prone to skidding when taking a turn. Hence vehicle centripetal acceleration is essential for safe turning. Hence centripetal force is provided by sloping the curved road. Due to sloping roads, the possibility of vehicle skidding is very less.

Suppose a vehicle of mass m is moving at speed v along a road of radius r. As shown in the figure, the slope of the road is θ along with the slope.

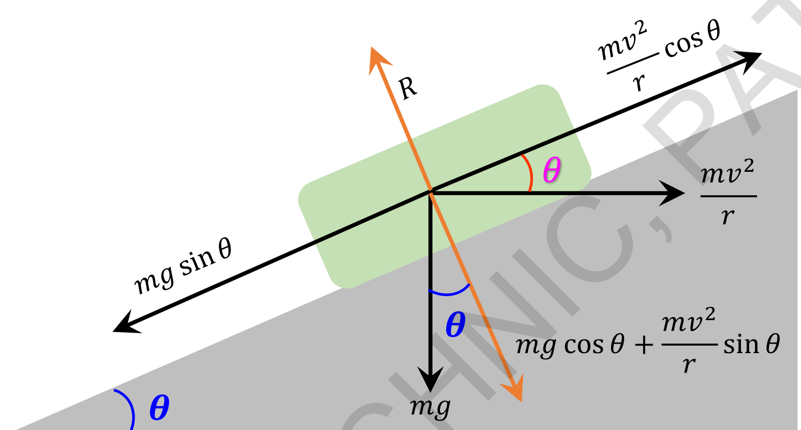
The weight force mg due to the mass of the vehicle always acts downwards and the centripetal force acts towards the center. The forward and vertical (perpendicular) components of both these forces and the perpendicular force N on the surface of the vehicle are equal to each other.

Negative component of centripetal force $=$ Negative component of gravitational force

$$\therefore \frac{mv^2}{r}\cos\,\theta\ = mg\sin\theta \ldots\ldots\ldots (1)$$

$$\therefore \frac{v^2}{rg} = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$



$$\therefore \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

The above equation gives the value of slope required for the road. Looking at the equation here it is seen that

1. The slope does not depend on the mass m of the vehicle.
2. As the value of safe speed on the road is to be increased, the value of slope should also be increased.

Maximum Safe Speed: Centripetal force is required when a two-wheeler takes a turn. The centripetal force required for this is due to the frictional force generated between the tires of the vehicle and the road.

If a vehicle of mass m is to move at a safe speed v on a forward road of radius r, then the vehicle

$$F_c = \frac{mv^2}{r}$$

this much centripetal force must be obtained.

Friction force between road and tire

$$F_f = \mu_s mg$$

Where μ_s is the coefficient of static friction between the road and the tyre.

Frictional force between road and tire is greater than centripetal force.

$$\mu_s mg > \frac{mv^2}{r}$$

If the maximum safe speed of the vehicle v_{max} is to be maintained,

$$\therefore \frac{m{v_{max}}^2}{r} = \mu_s mg$$

$$\therefore {v_{max}}^2 = \mu_s rg$$

$$\therefore v_{max} = \sqrt{\mu_s rg}$$

Remember: Maximum safe speed does not depend on vehicle or passenger mass m.

Also, the value of friction force is different for each type of road and also because the value of friction force is more uncertain it is extremely risky to travel at higher speed than v_{max}.

Bending of Cyclist:

Bending of the bicycle slightly inwards from the vertical position to reduce the reliance on frictional forces when turning on curved roads. By doing this, the required centripetal force is obtained.

Suppose,

$$Mass\ of\ cyclist = m$$

$$Speed\ of\ cyclist = v$$

$$Radius\ of\ circular\ path = r$$

$$Angle\ of\ inclination\ with\ upward\ direction = \theta$$

As shown in the figure, the weight of the cyclist acts downwards. Perpendicular force acting on the angle of inclination of the cyclist with the vertical direction.

The perpendicular force acting on the cyclist can be divided into two components. Its forward component provides the necessary centripetal force.

$$\therefore R\sin\theta = \frac{mv^2}{r} \ldots\ldots\ldots (1)$$

The upward component of the vertical force is equivalent to the cyclist's weight.

$$\therefore R\cos\theta = mg \ldots\ldots\ldots (2)$$

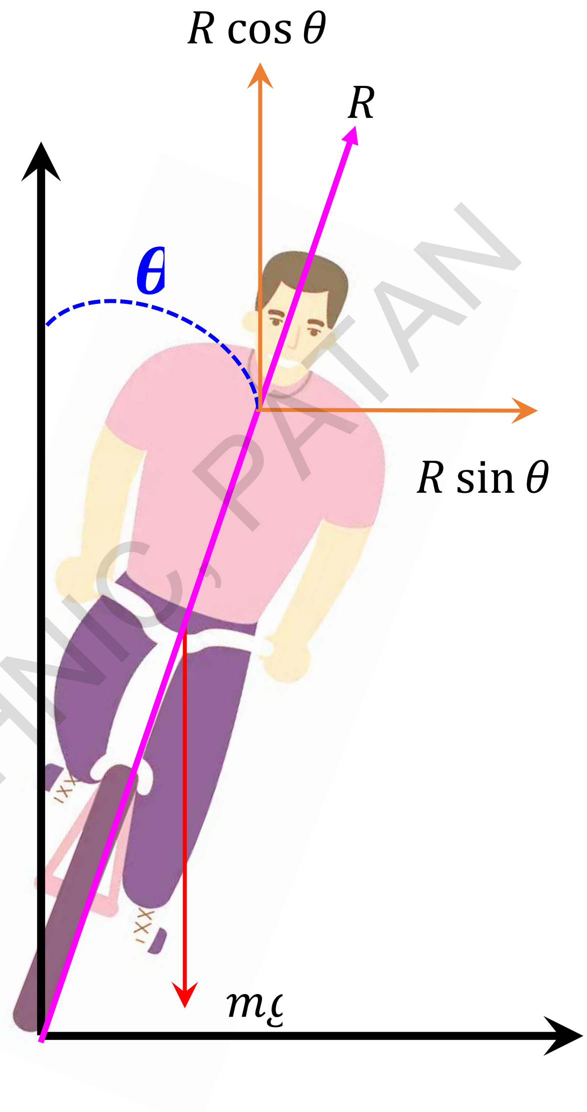
Taking the ratio of Eqs.(1) and (2),

$$\frac{R\sin\theta}{R\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\therefore \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{mrg}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$

$$\therefore \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$



Looking at the equation here it is seen that,

1. Bending angle does not depend on the mass m of the cyclist.
2. To increase the value of safe speed on the road, the value of bending angle should also be increased.