Diploma Engineering

Tutorial

Semester 2

Applied Mathematics - DI02000011

(As per NEP 2020)

[Branch_____________________]

Enrolment No	
Name	
Branch	
Academic Term	
Institute	

Directorate Of Technical Education

Gandhinagar - Gujarat

DTE's Vision:

To facilitate quality technical and professional education having relevance for both industry and society, with moral and ethical values, giving equal opportunity and access, aiming to prepare globally competent technocrats.

DTE's Mission:

- Quality technical and professional education with continuous improvement of all the resources and personnel
- To promote conducive ecosystem for Academic, Industry, Research, Innovations and Startups
- To provide affordable quality professional education with moral values, equal opportunities, accessibility and accountability
- To allocate competent and dedicated human resources and infrastructure to the institutions for providing world-class professional education to become a Global Leader ("Vishwa Guru")

Institute's Vision:**(Student should write)**

Institute's Mission:**(Student should write)**

Department's Vision:**(Student should write)**

Department's Mission:**(Student should write)**

Programme Outcomes (POs) :

1. **Basic and Discipline specific knowledge:** Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specialization to solve the *engineering* problems.

2. **Problem analysis**: Identify and analyse well-defined *engineering* problems using codified standard methods.

3. **Design/ development of solutions:** Design solutions for *engineering* well-defined technical problems and assist with the design of systems components or processes to meet specified needs.

4. **Engineering Tools, Experimentation and Testing:** Apply modern *engineering* tools and appropriate technique to conduct standard tests and measurements.

5. **Engineering practices for society, sustainability and environment:** Apply appropriate technology in context of society, sustainability, environment and ethical practices.

6. **Project Management:** Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.

7. **Life-long learning:** Ability to analyze individual needs and engage in updating in the context of technological changes *in field of engineering.*

Course Outcomes (COs):

1. Demonstrate the ability to Crack engineering related problems based on Matrices.
2. Demonstrate the ability to solve engineering related problems based on applications of differentiation.
3. Demonstrate the ability to solve engineering related problems based on applications of integration.
4. Develop the ability to apply differential equations to significant applied problems.
5. Solve applied problems using the concept of mean.

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Enrolment No: **Name:**

Term:

Date: ……………

Tutorial No.1
(Unit No.1: Matrices)

Solve simple problems using the concept of algebraic operations of matrices.

COURSE OUTCOME	Demonstrate the ability to Crack engineering related problems based on Matrices.

<u>List of main formulas/working rules:</u>

1	Order of a matrix with n rows and m columns is $\boldsymbol{n \times m}$.
2	The transpose of matrix $A = [a_{ij}]_{n\times m}$ (i.e. matrix A of order $n \times m$ with elements a_{ij} where, $i = 1,2,3,\dots,n$ and $j = 1,2,3,\dots,m$.) is $A^T = [a_{ji}]_{m\times n}$. e.g. Transpose of $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & -1 \end{bmatrix}_{2\times 3}$ is $A^T = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 5 & -1 \end{bmatrix}_{3\times 2}$
3	For matrices $A = [a_{ij}]_{n\times m}$and $= [b_{ij}]_{n\times m}$, $A \pm B = [a_{ij} \pm B_{ij}]_{n\times m}$**(Note that, addition and subtraction between matrices is possible if and only if they have same orders**.)
4	For any two matrices A and B, $A \times B$ is possible if and only if, $\boldsymbol{no. of\ columns\ in\ A = No. of\ rows\ in\ b}$. Also, the order of resulting matrix $A \times B$ is $\boldsymbol{no. of\ rows\ in\ A\ \times no. of\ columns\ in\ B}$.
5	Number of elements in a matrix $A = [a_{ij}]_{n\times m}$ is $\boldsymbol{nm}$.
6	For matrices A and B, • $(AB)^T = B^T A^T$ • $(A^T)^T = A$ • $(A + B)^T = A^T + B^T$ • $(k \cdot A)^T = k \cdot A^T, k \in R$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}1 & -1\\1 & 3\end{bmatrix}$ = ____.				U
	(a) $\begin{bmatrix}2 & 1\\4 & 7\end{bmatrix}$	(b) $\begin{bmatrix}-2 & 1\\4 & 7\end{bmatrix}$	(c) $\begin{bmatrix}2 & 1\\4 & 0\end{bmatrix}$	(d) $\begin{bmatrix}2 & -1\\4 & 7\end{bmatrix}$	
2	Order of $\begin{bmatrix}3 & 9 & 0\\1 & -2 & 0\end{bmatrix}$ =____.				U
	(a) 2×2	(b) 3×2	(c) 2×3	(d)3×3	
3	$\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix}\times\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix}$=_____.				A
	(a) $\begin{bmatrix}5 & 12\\10 & 6\end{bmatrix}$	(b) $\begin{bmatrix}9 & 8\\37 & 36\end{bmatrix}$	(c) $\begin{bmatrix}6 & 8\\7 & 7\end{bmatrix}$	(d)$\begin{bmatrix}-4 & -4\\3 & 4\end{bmatrix}$	
4	If $A=\begin{bmatrix}1\\2\end{bmatrix}$ and B = $[3 \quad 4]$ then $A\times B=$ ______.				U
	(a) $\begin{bmatrix}3 & 4\\6 & 8\end{bmatrix}$	(b) $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}$	(c) $[11]$	(d)$[-5]$	
5	$\begin{bmatrix}1 & 3 & 4 & 9\\2 & 3 & 0 & 4\\5 & 6 & 8 & 2\end{bmatrix}^T$ = _____.				U
	(a) $\begin{bmatrix}0 & -2 & 5\\3 & 0 & 6\\4 & 0 & 8\\9 & -4 & 2\end{bmatrix}$	(b) $\begin{bmatrix}1 & 2 & 5\\3 & 3 & 6\\4 & 0 & 8\\9 & 4 & 2\end{bmatrix}$	(c) $\begin{bmatrix}1 & 2 & 5\\3 & 3 & 6\end{bmatrix}$	(d)$\begin{bmatrix}1 & 3 & 4 & 9\\2 & 3 & 0 & 4\\5 & 6 & 8 & 2\end{bmatrix}$	
6	If $\begin{bmatrix}x-3 & 2\\4 & 0\end{bmatrix}=\begin{bmatrix}5 & 2\\4 & 0\end{bmatrix}$ then x = ____.				U
	(a) $x=0$	(b) $x=2$	(c) $x=-8$	(d)$x=8$	
7	If A is of order 3×2 and B is of order 2×1 then order of matrix AB is _____.				U
	(a) 2×2	(b) 2×1	(c) 3×1	(d)3×2	
8	If A is a square matrix then all the diagonal elements if $A-A^T$ are ____.				R
	(a) 2	(b) -1	(c) 0	(d)1	
9	If A is of order 3×4 and B is of order 4×2 then no. of elements in AB is___.				U
	(a) 6	(b) 5	(c) 8	(d)1	
10	If for a square matrix A, $A=-A^T$ then A is called ________ Matrix.				R
	(a) Symmetric	(b) Skew-Symmetric	(c) Singular	(d) Non-Singular	
11	For any matrix A, $(A^T)^T$=_____ .				R
	(a) A^T	(b) $-A^T$	(c) $-A$	(d)A	

12	If $\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} x-2 & 2 \\ 0 & y+1 \end{bmatrix}$, then $x =$ _____and $y =$ _____				U
	(a) -1 and -2	(b) 1 and -4	(c) 3 and -4	(d) 3 and -2	
13	If X + $\begin{bmatrix} 0 & -4 \\ 3 & -2 \\ 4 & 3 \end{bmatrix}$ = $\begin{bmatrix} 1 & 5 \\ -3 & 2 \\ -5 & 4 \end{bmatrix}$ then X = _____.				U
	(a) $\begin{bmatrix} 1 & 9 \\ -6 & 4 \\ -9 & 1 \end{bmatrix}$	(b) $\begin{bmatrix} -1 & -9 \\ 6 & -4 \\ 9 & -1 \end{bmatrix}$	(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}$	(d) $\begin{bmatrix} -1 & 9 \\ 6 & 4 \\ 9 & 1 \end{bmatrix}$	
14	A matrix A is called a _____ matrix if $\det(A) = 0$.				R
	(a) Symmetric	(b) Skew-Symmetric	(c) Singular	(d) Non-Singular	
15	If $\begin{bmatrix} 2x-3 & x-5 \\ -3 & 5 \end{bmatrix}$ is a symmetric matrix then $x =$_____.				U
	(a) $x = 0$	(b) $x = 2$	(c) $x = -8$	(d) $x = 8$	

Q.2<u>Do as directed (3 0R 4 MARKS QUESTIONS):</u>

1	Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix}$, find $2A - 4B + C$.	U

2	If $M = \begin{bmatrix} -2 & 3 & 8 \\ 5 & -7 & 9 \\ 1 & -4 & 6 \end{bmatrix}$ and $N = \begin{bmatrix} 15 & -6 & 2 \\ 11 & 4 & 7 \\ 13 & 5 & 6 \end{bmatrix}$ then prove that $(M+N)^T = M^T + N^T$.	A
3	A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$ then prove that, $A^T B^T = (BA)^T$.	A

4	If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then find $+A^T + I$.	A
5	If $\begin{bmatrix} x+3 & -6 & 2 \\ y+1 & 2 & 0 \\ z-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -6 & 2 \\ -3 & 2 & 0 \\ 4x & -21 & 0 \end{bmatrix}$ then find the values of x , y & z .	A

6	If $A = \begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}$ then find $A^2 + I$.	A
7	Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 1 \\ 1 & 7 & 5 \end{bmatrix}$, find AB.	A

8	If $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ then , prove that AB = AC.	A
9	For $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$ find AB and BA whichever is possible.	A

10	If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ then find AB.	A

Answer Key:

Q-1: Answers

1)	**(a)**	2)	**(c)**	3)	**(b)**	4)	**(a)**	5)	**(b)**
6)	**(d)**	7)	**(c)**	8)	**(c)**	9)	**(a)**	10)	**(b)**
11)	**(d)**	12)	**(c)**	13)	**(a)**	14)	**(c)**	15)	**(b)**

Q-2: Answers

1)	$\begin{bmatrix} 13 & 12 & 7 \\ 2 & 7 & 3 \\ 13 & 16 & 8 \end{bmatrix}$	4)	$\begin{bmatrix} 3 & 1 \\ 1 & 7 \end{bmatrix}$
5)	$x = -3, y = -4,$ $z = -9$	6)	$A^2 + I = \begin{bmatrix} -2 & 6 \\ -6 & 13 \end{bmatrix}$
7)	$\begin{bmatrix} -2 & 7 & 5 \\ 12 & 17 & 10 \end{bmatrix}$	9)	AB is not possible, $BA = \begin{bmatrix} 2 & 3 & 5 \\ 8 & 2 & 18 \end{bmatrix}$
10)	$AB = \begin{bmatrix} -2 & -2 \\ 0 & -6 \end{bmatrix}$		

Link of BISAG Lectures

YouTube Channel name: DTEGUJ (Link: https://www.youtube.com/@dteguj8385) (Directorate of Technical Education Department, Government of Gujarat)	
Basic concepts of matrices, Addition, Subtraction, Multiplication and Transpose.	
1	https://www.youtube.com/watch?v=iULS7nE4v94&t=1724s
2	https://www.youtube.com/watch?v=TAAo3vEo3d8&t=2411s

Suggested Activities and website list for aspiring students

- https://www.mathsisfun.com/algebra/matrix-multiplying.html
- https://www.mathsisfun.com/algebra/matrix-calculator.html
- https://www.programiz.com/python-programming/examples/add-matrix

Date: …………..

Tutorial No.2
(Unit No.1: Matrices)

Use the concept of adjoint of a matrix to find the inverse of a matrix.

COURSE OUTCOME	Demonstrate the ability to Crack engineering related problems based on Matrices.

List of main formulas/working rules:

1	**Minor:** • Every element of a square matrix has a unique minor. • The minor of element of matrix is the determinant obtained by eliminating the row and the column containing that particular element. For example, ➢ In the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ the minor of $a_{11} = a_{22}$. ➢ In the matrix $B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, the minor of $a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$ ➢ In a similar way we can find the minors for all the elements of the matrix.
2	**Sign of Cofactor:** The sign of cofactor of each element of a matrix is obtained using below formula. Sign of cofactor of $a_{ij} = (-1)^{i+j}$. For example, ➢ Sign of cofactor of $a_{21}$is $(-1)^{2+1} = (-1)^3 = -1$which indicates that the sign is negative. ➢ Sign of cofactor of a_{33} is $(-1)^{3+3} = (-1)^6 = 1$which indicates that the sign is positive.
3	**Cofactors:** • The cofactor of a_{ij} is denoted by A_{ij}. • Formula: A_{ij} = Sign of cofactor of a_{ij} $\times$ Minor of a_{ij}.

4	**Adjoint of a matrix:** • The transpose of the cofactor matrix of the square matrix is called the **adjoint of the matrix.** • **The adjoint of matrix** $A = [a_{ij}]_{n\times n}$ **is denoted by** $adj(A)$**.** $adj(A) = [A_{ij}]^T_{n\times n}$ Where, A_{ij} are the cofactors of the elements a_{ij}. • For any square matrix A, $A \cdot adj(A) = adj(A) \cdot A = \|A\| \cdot I$. Where I is identity matrix of order same as order of matrix A.
5	**Inverse of a matrix:** • The inverse of matrix A exists if and only if $\det(A) \neq 0$. • The inverse of matrix A is denoted by A^{-1}. • Formula: $A^{-1} = \frac{adj(A)}{\|A\|}$. (Remember: $\|A\| = \det(A)$.)

Q.1Do as directed (ONE MARK QUESTIONS):

1	For matrix A , If A^{-1} exist then $A \times A^{-1} =$ ______.				R
	(a) I	(b) 0	(c) $-A$	(d) $-I$	
2	The Adjoint of $\begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix}$=______.				U
	(a) $\begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix}$	(b) $\begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$	(c) $\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$	(d) $\begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix}$	
3	For any square matrix A of order 2, $adj(adj(A))$= ______				U
	(a) A	(b) A^T	(c) A^{-1}	(d) $adj(A)$	
4	For any square matrix A, if $A^3 + 2A^2 - 2A + 3I = 0$, then A^{-1} =______.				A
	(a)$A^2 + 2A - 2I$	(b) $\frac{1}{3}(A^2 + 2A - 2I)$	(c) $-\frac{1}{3}(A^2 + 2A - 2I)$	(d)$-(A^2 + 2A - 2I)$	
5	In the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ 4 & 2 & -2 \end{bmatrix}$, the cofactor of 3 (i.e. A_{21}) = _____.				U
	(a) -1	(b) 1	(c) 2	(d) 0	
6	For any square matrix A, $(A^{-1})^{-1}$ = ______.				R
	(a) A^{-1}	(b) A	(c) $-A$	(d) $adj(A)$	

7	The inverse of matrix $\begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$ = _______.				A
	(a) $\begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$	(b) $\begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}$	(c) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 \end{bmatrix}$	(d) $\begin{bmatrix} -1 & 1 \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$	
8	The sign of cofactor of the element a_{32} =_______.				U
	(a) $\pm$	(b) $+$	(c) $-$	(d) Undefined	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

1	If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then show that A. A^{-1} = I.	A

2	For A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$ then find A^{-1}.	A
3	If A=$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 5 \end{bmatrix}$ and B =$\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$ find $(\mathrm{A}+\mathrm{B})^{-1}$	A

4	Find the inverse of $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$.	A
5	For $A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$, Show that $(AB)^{-1} = B^{-1}A^{-1}$.	A

6	If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then prove that Adj A = A.	A
7	If A=$\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ then find matrix B, such that AB = BA = I.	A

8	If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then prove that $3A^{-1} = A^T$.	A

Answer Key:

Q-1: Answers

1)	**(a)**	2)	**(b)**	3)	**(a)**	4)	**(c)**	5)	**(d)**
6)	**(b)**	7)	**(c)**	8)	**(c)**				

Q-2: Answers

2)	Hint: Use $7A^{-1} = 5I - A$ $A^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$	3)	$(A+B)^{-1} = \begin{bmatrix} \frac{7}{11} & \frac{3}{11} & -\frac{6}{11} \\ -\frac{10}{11} & \frac{2}{11} & \frac{7}{11} \\ -\frac{1}{11} & -\frac{2}{11} & \frac{4}{11} \end{bmatrix}$
4)	$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{9}{2} & -\frac{7}{2} & \frac{11}{2} \\ -\frac{5}{2} & -\frac{5}{2} & \frac{7}{2} \end{bmatrix}$	7)	$B = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

Link of BISAG Lectures

YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)	
Basic concepts of matrices, Addition, Subtraction, Multiplication and Transpose.	
1	https://www.youtube.com/watch?v=I2Jdvo8ZAmE&t=1849s
2	https://www.youtube.com/watch?v=geQY5avNew4&t=196s

Suggested Activities and website list for aspiring students

- https://www.mathsisfun.com/algebra/matrix-inverse.html

Date: ……………

Tutorial No.3
(Unit No.1: Matrices)

Solve system of linear equations using matrices. Use suitable software to demonstrate the geometric meaning of solution of system of linear equations.

COURSE OUTCOME	Demonstrate the ability to Crack engineering related problems based on Matrices.

List of main formulas/working rules:

1	**Geometric interpretation and meaning of solution:** A linear equation with two variables represents a line in the xy-plane. The solution of system of linear equations with two variables gives the point of intersection of the lines present in the system.
2	**Solution of system of linear equations:** $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$ **Step-1:** Convert the system of linear equations in to a matrix equation. $AX = B$ Where, $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -c_1 \\ -c_2 \end{bmatrix}$ **Step-2:** Check whether A is a non-singular matrix or not. (i.e. $\|A\| \neq 0$) (Note: The system has unique solution if and only if A is a non-singular matrix.) If $\|A\| \neq 0$, then go to step-3. **Step-3:** Find A^{-1}. **Step-4:** $AX = B$ implies, $X = A^{-1}B$. Hence, compute $A^{-1}B$ which will give the solution.

Q.1 Do as directed (ONE MARK QUESTIONS):

1	Do the system of linear equation $x + y + 1 = 0, 3x + 3y - 7 = 0$ has a unique solution?				U
	(a) Yes	(b) No			
2	If the linear equations in a system with two variables represent parallel lines then the system has a unique solution				U
	(a) False	(b) True	(c)Uncertain		
3	The system of linear equations $4x - 6y + 1 = 0, ax - 3y = 0$ do not have unique solution if a= ______.				U
	(a)$a = 0$	(b)$a = 1$	(c)$a = 2$	(d) $a = -2$	
4	The solution of the system of linear equation $x = 0, 2y = 4$ is the point _______.				U
	(a)$(2,0)$	(b)$(-2,0)$	(c) $(0,2)$	(d) $(0,-2)$	
5	The matrix equation representing the system of linear equations, $2x + 4y - 3 = 0,\ 5x + 2y + 2 = 0$, is ______.				U
	(a)$\begin{bmatrix} 2 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$	(b)$\begin{bmatrix} -2 & 4 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$	(c)$\begin{bmatrix} 5 & -2 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$	(d) $\begin{bmatrix} 2 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

1	Solve the system of linear equations $3x + y = 9\ , 2x - 3y = \ -5$ using matrices.	A

2	Solve the system of linear equations $2x + 3y = 1,\ y - 4x = 2$ using matrices.	A
3	Solve the system of linear equations $\frac{3}{x} + \frac{2}{y} + 2 = 0, \frac{1}{x} - \frac{4}{y} + 6 = 0$ using matrices.	A

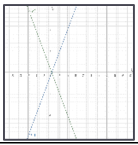
4	Solve the system of linear equations $2x + 3y = 6xy$ and $x - y = xy$ using matrices.	A
5	Solve the system of linear equations $x + y = 0, x - y = 0$ using matrices and show it's geometrical interpretation using GeoGebra	

Answer Key:

Q-1: Answers

1)	**(b)**	2)	**(a)**	3)	**(c)**	4)	**(c)**	5)	**(d)**

Q-2: Answers

1)	$x = 2, y = 3$	2)	$x = -\frac{5}{14}, y = \frac{4}{7}$
3)	$x = -\frac{7}{10}, y = \frac{7}{8}$	4)	$x = \frac{5}{4}, y = \frac{5}{9}$
5)	$\boldsymbol{x = 0, y = 0}$ **Link of GeoGebra Graphing Calculator: https://www.geogebra.org/graphing?lang=en** 		

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Basic concepts of matrices, Addition, Subtraction, Multiplication and Transpose.	
1	https://www.youtube.com/watch?v=I2Jdvo8ZAmE&t=1849s
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Suggested Activities and website list for aspiring students

- https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html

Date: ……………

Tutorial No. 4
(Unit No.2: Differentiation and its Applications)

Solve examples related to 1st rule of derivative, working rules.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of differentiation

List of main formulas/working rules:

<table>
<tr><td rowspan="2">1</td><td colspan="2">Definition of Derivative</td></tr>
<tr><td colspan="2">Derivative of function $f(x)$ denoted as $f'(x)$ or $\frac{d}{dx}f(x)$ and defined as below
$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $f'(x)=\lim_{t\to x}\frac{f(t)-f(x)}{t-x}$</td></tr>
<tr><td rowspan="7">2</td><td colspan="2">Some standard formulas:</td></tr>
<tr><td>1) $\frac{d}{dx}k=0$, where k is a constant.</td><td>2) $\frac{d}{dx}x^n=n\,x^{n-1}$</td></tr>
<tr><td>3) $\frac{d}{dx}a^x=a^x\log_e a$</td><td>4) $\frac{d}{dx}e^x=e^x$</td></tr>
<tr><td>5) $\frac{d}{dx}\sin x=\cos x$</td><td>6) $\frac{d}{dx}\cos x=-\sin x$</td></tr>
<tr><td>7) $\frac{d}{dx}\tan x=\sec^2 x$</td><td>8) $\frac{d}{dx}\cot x=-\operatorname{cosec}^2 x$</td></tr>
<tr><td>9) $\frac{d}{dx}\sec x=\sec x\cdot\tan x$</td><td>10) $\frac{d}{dx}\operatorname{cosec} x=-\operatorname{cosec} x\cdot\cot x$</td></tr>
<tr><td>11) $\frac{d}{dx}\log_e x=\frac{1}{x}$</td><td></td></tr>
<tr><td rowspan="2">3</td><td colspan="2">Working Rules of Derivative</td></tr>
<tr><td colspan="2">1) $\frac{d}{dx}\left(f(x)\pm g(x)\right)=\frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)$

2) $\frac{d}{dx}\left(kf(x)\right)=k\frac{d}{dx}f(x)$, k is a constant

3) $\frac{d}{dx}\left(f(x)\cdot g(x)\right)=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$, (Product rule)

4) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{\left(g(x)\right)^2}$, $g(x)\neq 0$ (Quotient rule)</td></tr>
</table>

Q.1 Do as directed (ONE MARK QUESTIONS):

1	$\frac{d}{dx}(\sin^2 x + \cos^2 x) =$ ______				U
	(a) 1	(b) 0	(c) $\sin x$	(d) $\cos x$	
2	$\frac{d}{dx}(34) =$ ______				R
	(a) 1	(b) 33	(c) 0	(d) 35	
3	$\frac{d}{dx}\left(\tan x - 2^x\right) =$ ______				U
	(a) $\sec^2 x$	(b) $\sec^2 x - 2^x \log_e 2$	(c) $\sin x$	(d) $\cos x$	
4	$y = 5e^x - 4$ then $\frac{dy}{dx} =$ _____.				U
	(a) $5e^x - 4$	(b) $5e^x$	(c) $e^x - 4$	(d) $5e^x - 3$	
5	$f(x) = x^2 + 2x - 11$ then $f'(0) =$ ______.				U
	(a) 1	(b) 11	(c) 2	(d) 0	
6	$y = \sqrt{x}$ then $y' =$ _____.				U
	(a) $\frac{1}{2\sqrt{x}}$	(b) $\sqrt{x}$	(c) x	(d) 1	
7	$\frac{d}{dx}(\log x) =$ ______				R
	(a) x	(b) $\frac{1}{x}$	(c) $\sin x$	(d) $\cos x$	
8	$\frac{d}{dx}(\log 4) =$ ______				U
	(a) 4	(b) $\frac{1}{4}$	(c) 0	(d) $\log x$	
9	If $y = \sec^2 x - \tan^2 x$ than $\frac{dy}{dx} =$ _____.				U
	(a) 1	(b) 0	(c) $\tan x$	(d) $\sec x$	
10	$f(x) = 3^x$ then $f'(1) =$ ______				A
	(a) 3	(b) $\frac{1}{3}$	(c) $3\log_e 3$	(d) $\log_e 3$	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

1	Find the derivative of following functions using **definition of derivative.** **(1)** $f(x)=x^2$ **(2)** $f(x)=e^x$ **(3)** $f(x)=\sin x$ **(4)** $f(x)=\log x$	A

2	Find the derivative of following functions using 1^{st} **rule of derivative.** **(1)** $f(x) = 2x^3 + 4$ **(2)** $f(x) = 5^x$ **(3)** $f(x) = \cos x$	A
3	Find $\frac{d}{dx}\left(4x^3 + 2x^2 - 3\right)$	A

4	Find $\frac{d}{dx}\left(x^3+3^x-3\right)$	A
5	Find $\frac{d}{dx}\left(\frac{2x^3-3x^2-3}{x}\right)$	A

6	Find $\frac{d}{dx}\left(e^x \sin x\right)$	A
7	Find $\frac{d}{dx}\left(x^2 \log x\right)$	A

8	Find $\frac{d}{dx}\left(2x^3 cox\right)$	A
9	Find $\frac{d}{dx}\left(3^x \tan x + 2\right)$	A

10	Find $\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)$	A
11	Find $\frac{d}{dx}\left(\frac{\log x}{x}\right)$	A

12	For $y = \dfrac{1-\sin x}{1+\sin x}$, find y'.	A
13	For $f(x) = \dfrac{\tan x}{x}$, find $f'(x)$.	A

14	Find $\frac{d}{dx}\left(\frac{a+b\sin x}{a\sin x+b}\right)$	A
15	Find the derivative of $f(x)=2x\sin x-x^3\cos x$	A

Answer Key:

Q-1: Answers

1)	**(b)**	2)	**(c)**	3)	**(b)**	4)	**(b)**	5)	**(c)**
6)	**(a)**	7)	**(b)**	8)	**(c)**	9)	**(b)**	10)	**(c)**

Q-2: Answers

1)	(1) $2x$	(2) e^x	(3) $\cos x$	(4) $\frac{1}{x}$
2)	(1) $6x^2$	(2) $5^x \log_e 5$	(3) $-\sin x$	

3)	$12x^2 + 4x$	4)	$3x^2 + 3^x \log_e 3$
5)	$4x - 3 + \frac{3}{x^2}$	6)	$e^x(\sin x + \cos x)$
7)	$x + 2x \log x$	8)	$-2x^3 \sin x + 6x^2 \cos x$
9)	$3^x(\sec^2 x + \log_e 3 \tan x)$	10)	$\frac{4x}{\left(x^2+1\right)^2}$
11)	$\frac{1-\log x}{x^2}$	12)	$\frac{2\cos x}{(1+\sin x)^2}$
13)	$\frac{x\sec^2 x - \tan x}{x^2}$	14)	$\frac{\left(b^2-a^2\right)\cos x}{(a\sin x + b)^2}$
15	$2x\cos x + 2\sin x + x^3 \sin x - 3x^2 \cos x$		

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Differentiation and Its applications
1	https://bit.ly/2IYAMip
2	https://bit.ly/2IYAS9L

Suggested Activities and website list for aspiring students

- GEOMETRICAL MEANING OF DERIVATIVE : https://www.geogebra.org/m/DbZh24kJ
- https://archive.nptel.ac.in/courses/111/106/111106146/
- https://www.whitman.edu/mathematics/calculus_online/chapter03.html

Date: ……………

Tutorial No. 5
(Unit No. 2: Differentiation and its Applications)

Solve examples of derivative related to Chain Rule, Implicit functions.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of differentiation

List of main formulas/working rules:

1	**Chain Rule (Differentiation of composite function)** Let $y = gof$ be a real valued function which is a composite of two functions g and f. $y = (gof)x = g(f(x)) = g(u)$ Suppose u= $f(x)$ $\therefore \frac{dy}{dx} = g'(u) \cdot f'(x)$ $\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	
2	**Some standard formulas:**	
	1) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	2) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
	3) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	4) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
	5) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\|x\|\sqrt{x^2-1}}$	6) $\frac{d}{dx}(co\sec^{-1} x) = \frac{-1}{\|x\|\sqrt{x^2-1}}$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	$y = e^{2x}$ then $\frac{dy}{dx} =$ ____.				U
	(a) 1	(b) $\frac{e^{2x}}{2}$	(c) e^{2x}	(d) $2e^{2x}$	

2	$y=\sin 3x$ then $\frac{dy}{dx}=$ _____.				A
	(a) $\frac{\cos 3x}{3}$	(b) $3\cos 3x$	(c) $\sin 3x$	(d) $\cos 3x$	
3	$y=\log(2x-1)$ then $\frac{dy}{dx}=$ _____.				A
	(a) $\frac{1}{2x-1}$	(b) $\frac{2}{2x-1}$	(c) $\frac{1}{x-1}$	(d) $\cos x$	
4	$xy=1$ then $\frac{dy}{dx}=$ _____.				A
	(a) $\frac{-1}{x^2}$	(b) $\frac{1}{x^2}$	(c) 1	(d) 0	
5	$\frac{d}{dx}(\cos^{-1}x)=$ ____				R
	(a) 1	(b) 0	(c) $\frac{-1}{\sqrt{1-x^2}}$	(d) $\frac{1}{\sqrt{1-x^2}}$	
6	$\frac{d}{dx}(\tan^{-1}x)=$ ____				R
	(a) $\frac{1}{\sqrt{1-x^2}}$	(b) $\frac{-1}{\sqrt{1-x^2}}$	(c) $\frac{1}{1+x^2}$	(d) $\frac{-1}{1+x^2}$	
7	$\frac{d}{dx}(\sin^{-1}x+\cos^{-1}x)=$ ____				U
	(a) 1	(b) 0	(c) $\frac{-1}{\sqrt{1-x^2}}$	(d) $\frac{1}{\sqrt{1-x^2}}$	
8	$\frac{d}{dx}(\tan^{-1}x+\cot^{-1}x)=$ ____				U
	(a) 1	(b) 0	(c) $\frac{1}{1+x^2}$	(d) $\frac{-1}{1+x^2}$	
9	If $y=e^{\tan x}$ then $\frac{dy}{dx}=$ _____.				A
	(a) $e^{\tan x}\tan x$	(b) $e^{\tan x}\tan^2 x$	(c) $e^{\tan x}\sec^2 x$	(d) $e^{\tan x}\sec x$	
10	$y=\log(5x)$ then $\frac{dy}{dx}=$ _____.				A
	(a) $\frac{1}{5}$	(b) $\frac{5}{x}$	(c) $\frac{1}{5x}$	(d) $\frac{1}{x}$	

Q.2Do as directed (3 0R 4 MARKS QUESTIONS):

1	Find $\frac{d}{dx}\left(e^{\sin x}\right)$	A
2	Find $\frac{d}{dx}\left(\sin(x^2)\right)$	A

3	Find $\frac{d}{dx}\left(\sin^2 x\right)$	A
4	Find $\frac{d}{dx}\log\left(x^2+1\right)$	A

5	Find $\frac{d}{dx}\log\left(\sqrt{x^2+a^2}\right)$	A
6	Find $\frac{d}{dx}\left(e^{4x}\cos 3x\right)$	A

7	Prove that $\frac{d}{dx}\left(\frac{\sin(\log x)}{x}\right)=\frac{1}{x^2}\left[\cos(\log x)-\sin(\log x)\right]$	A
8	Prove that $\frac{d}{dx}\log\left(\sec x+\tan x\right)=\sec x$	A

9	Prove that $\frac{d}{dx}\log\left(\frac{\sin x}{1+\cos x}\right)=\cos ec\ x$	A
10	If $x^3+y^3=3axy$ then find $\frac{dy}{dx}$.	

11	If $x\cos y + y\cos x + 3 = 0$ then find $\frac{dy}{dx}$.	A
12	If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then find $\frac{dy}{dx}$.	A

13	If $y=\sin(x+y)$ then prove that $\frac{dy}{dx}=\frac{\cos(x+y)}{1-\cos(x+y)}$.	A
14	If $x+y=\sin(xy)$ then prove that $\frac{dy}{dx}=\frac{y\cos(xy)-1}{1-x\cos(xy)}$.	A

15	If $e^x + e^y = e^{x+y}$ then find $\frac{dy}{dx}$.	A

Answer Key:

Q-1: Answers

1)	**(d)**	2)	**(b)**	3)	**(b)**	4)	**(a)**	5)	**(c)**
6)	**(c)**	7)	**(b)**	8)	**(b)**	9)	**(c)**	10)	**(d)**

Q-2: Answers

1)	$\cos x \cdot e^{\sin x}$	2)	$2x\cos(x^2)$
3)	$2\cos x \sin x$	4)	$\frac{2x}{x^2+1}$
5)	$\frac{x}{x^2+a^2}$	6)	$e^{4x}(4\cos 3x - 3\sin 3x)$
10)	$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$.	11)	$\frac{dy}{dx} = \frac{y\sin x - \cos y}{\cos x - x\sin y}$
12)	$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$.	15)	$\frac{dy}{dx} = -e^{y-x}$

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Differentiation and Its applications
1	https://bit.ly/2x5QlSM
2	https://bit.ly/2QvIULE

Suggested Activities and website list for aspiring students

- https://www.whitman.edu/mathematics/calculus_online/chapter03.html

Date: ……………

Tutorial No. 6
(Unit No. 2: Differentiation and its Applications)

Solve the examples derivative of Parametric functions and second order derivative of simple functions.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of differentiation

List of main formulas/working rules:

1	**Differentiation of Parametric functions** Let parametric equation is given by $x = f(t)$ and $y = g(t), t \in [a,b]$ then $\frac{dx}{dt} = f'(t)$ and $\frac{dy}{dt} = g'(t)$ therefore $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}, f'(t) \neq 0$
2	**Higher Order Derivatives** Second order derivative of $f(x)$ is denoted as $f''(x) = \frac{d^2 f}{dx^2}$ and defined as $f''(x) = \frac{d^2 f}{dx^2} = \frac{d}{dx}\left(f'(x)\right)$ and when $y = f(x)$ then second derivative is dented as $y'' = y_2 = \frac{d^2 y}{dx^2}$ and defined as $y'' = y_2 = \frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	If $x = a\cos\theta, y = a\sin\theta$, then $\frac{dy}{dx} =$ ____.				A
	(a) $-\cot\theta$	(b) $\cot\theta$	(c) $\tan\theta$	(d) $\cos\theta$	
2	If $x = \tan\theta, y = \sec\theta$, then $\frac{dy}{dx} =$ ____.				A
	(a) $-\cot x$	(b) $\sin\theta$	(c) $\sec\theta$	(d) $\cos\theta$	
3	If $x = t^2 + 3, y = 3t^2 - 5$, then $\frac{dy}{dx} =$ ____.				A
	(a) 1	(b) 3	(c) 2	(d) 6	
4	$y = \sin 3x$ then $\frac{d^2y}{dx^2} =$ _____.				U
	(a) $-9\sin 3x$	(b) $9\sin 3x$	(c) $3\sin 3x$	(d) $3\cos 3x$	
5	$f(x) = x^3 + 2x - 9$ then $f''(2) =$ ______.				A
	(a) 1	(b) 0	(c) -6	(d) 12	
6	$\frac{d^2}{dx^2}(\cos x) =$ ____				U
	(a) 1	(b) $\sin x$	(c) $-\sin x$	(d) $-\cos x$	
7	$\frac{d^2}{dx^2}(\log x) =$ ____				U
	(a) $\frac{1}{x}$	(b) $\frac{-1}{x}$	(c) $\frac{1}{x^2}$	(d) $\frac{-1}{x^2}$	
8	$\frac{d^2}{dx^2}(5\sin x) =$ ____.				U
	(a) 5	(b) $5\cos x$	(c) $-5\sin x$	(d) $-5\cos x$	
9	$\frac{d^2}{dx^2}(\sin^2 x + \cos^2 x) =$ ______				U
	(a) 1	(b) 0	(c) $\sin x$	(d) $\cos x$	
10	$\frac{d^2}{dx^2}(2023) =$ ____				U
	(a) 1	(b) 0	(c) 2021	(d) 2022	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

1	For $x = at, y = \frac{a}{t}$, prove that $\frac{dy}{dx} = \frac{-y}{x}$	A
2	For $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ find $\frac{dy}{dx}$	A

3	For $x = a(1+\cos\theta), y = b(\theta+\sin\theta)$ find $\frac{dy}{dx}$	A
4	For $x = a\cos^2\theta, y = b\sin^2\theta$, prove that $\frac{dy}{dx} = \frac{-b}{a}$.	A

5	For $x = a\sin^3 t, y = b\cos^3 t$, prove that $\frac{dy}{dx} = \frac{-b}{a}\cot t$.	A
6	Find $\frac{d}{dx}(x)^x$	A

7	Find $\frac{d}{dx}(\sin x)^x$	A
8	Find $\frac{d}{dx}x^{\sin x}$	A

9	Find $\frac{d}{dx}(\sin x)^{\tan x}$	A
10	Find $\frac{d^2}{dx^2}(e^{5x}+11)$	A

11	For $x = at^2, y = 2at$, then find $\frac{d^2y}{dx^2}$	A
12	For $y = e^{2x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$	A

13	For $y = 2e^{3x} + 3e^{-2x}$, prove that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 6y = 0.$	A
14	For $y = A\cos pt + B\sin pt$, prove that $\frac{d^2y}{dt^2} + p^2y = 0.$	A

15	For $y = \log(\sin x)$, prove that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0.$	A

Answer Key:

Q-1: Answers

1)	**(a)**	2)	**(b)**	3)	**(b)**	4)	**(a)**	5)	**(d)**
6)	**(d)**	7)	**(d)**	8)	**(c)**	9)	**(b)**	10)	**(b)**

Q-2: Answers

2)	$\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}.$	3)	$\frac{dy}{dx} = -\frac{b}{a}\cot\frac{\theta}{2}.$
6)	$(x)^x (x + \log x)$	7)	$(\sin x)^x (x\cot x + \log\sin x)$
8)	$x^{\sin x}\left(\frac{\sin x}{x} + (\log x)\cos x\right)$	9)	$(\sin x)^{\tan x} (1+ \ \sec^2 x \cdot \log\sin x)$
10)	$25e^{5x}$	11)	$\frac{d^2y}{dx^2} = \frac{2y}{x^2}$

Link of BISAG Lectures

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	Differentiation and Its applications
1	https://bit.ly/2QvIULE
2	https://bit.ly/2QsO4bk
3	https://bit.ly/2U0S44F

Suggested Activities and website list for aspiring students

- https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_(Apex)/09%3A_Curves_in_the_Plane/9.02%3A_Parametric_Equations
- https://wiki.geogebra.org/en/Curves
- https://www.whitman.edu/mathematics/calculus_online/chapter03.html
- https://www.whitman.edu/mathematics/calculus_online/chapter06.html

Date: ……………

Tutorial No. 7
(Unit No. 2: Differentiation and its Applications)

Use concept of derivative to solve the problems related to velocity, acceleration and Maxima-Minima of given simple functions. Use suitable graphical software to visualize the concept of maxima-minima of function.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of differentiation

List of main formulas/working rules:

1	**Velocity and Acceleration** Equation of motion of a moving particle is given by $s = f(t)$, then velocity $v = \frac{ds}{dt}$ and acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.
2	**Maximum and Minimum values of function.** Steps to find maximum and minimum value of function $y = f(x)$ 1) Find $f'(x) = \frac{dy}{dx}$ and $f''(x) = \frac{d^2y}{dx^2}$ 2) Solve the equation $f'(x) = \frac{dy}{dx} = 0.$ Let $x_1, x_2, ..., x_n$ are solution of $f'(x) = 0.$ 3) Find values of $f''(x_1), f''(x_2), ..., f''(x_n)$ (4) If $f''(x_i) < 0$, then $y = f(x)$ has maximum value at $x = x_i$ and maximum value is $f(x_i)$. (5) If $f''(x_i) > 0$, then $y = f(x)$ has minimum value at $x = x_i$ and minimum value is $f(x_i)$.

Q.1 Do as directed (ONE MARK QUESTIONS):

1	Equation of motion of a moving particle is given by $s = f(t)$ then velocity $v =$___.				R
	(a) $f(t)$	(b) 0	(c) $\frac{ds}{dt}$	(d) $\frac{d^2s}{dt^2}$	
2	Equation of motion of a moving particle is given by $s = f(t)$ then acceleration $a =$___.				U
	(a) $f(t)$	(b) 0	(c) $\frac{ds}{dt}$	(d) $\frac{d^2s}{dt^2}$	
3	Equation of motion of a moving particle is given by $s = t\ \ +11$, then velocity at t=6 is __				A
	(a) 1	(b) 11	(c) 6	(d) 5	
4	$f(x)$ has maxima at $x = a$ if _____				R
	(a) $f'(a)=0, f''(a)=0$	(b) $f'(a)=0, f''(a)>0$	(c) $f'(a)>0, f''(a)<0$	(d) $f'(a)=0, f''(a)<0$	
5	The maximum value of a function $f(x)=\cos x$ is ___				U
	(a) 1	(b) 0	(c) -1	(d) 2	
6	The maximum value of a function $f(x)=\sin x$ is ___				U
	(a) 1	(b) 0	(c) -1	(d) 2	
7	$f(x)$ has minima at $x = a$ if _____.				R
	(a) $f'(a)=0, f''(a)=0$	(b) $f'(a)=0, f''(a)>0$	(c) $f'(a)>0, f''(a)<0$	(d) $f'(a)=0, f''(a)<0$	
8	Equation of motion of a moving particle is given by $s = f(t)$, particle change its direction at $t = 3$ seconds then velocity at $t = 3$ seconds is ____				A
	(a) 0	(b) 3	(c) 6	(d) 5	
9	Equation of motion of a moving particle is given by $s = 3t + 15$,then acceleration at t=4 is __.				A
	(a) 1	(b) 0	(c) 4	(d) 15	
10	Equation of motion of a moving particle is given by $s = f(t)$, v is velocity then acceleration $a =$___.				U
	(a) $f(t)$	(b) $\frac{dv}{dt}$	(c) $\frac{ds}{dt}$	(d) v	

Q.2Do as directed (3 0R 4 MARKS QUESTIONS):

1	Equation of motion of a moving particle is given by $s = 2t^3 + 3t^2 - 12t + 5$, find velocity at t=1 second and acceleration at t=2 second.	A
2	Equation of motion of a moving particle is given by $s = t^3 - 3t^2 + 4t + 3$, find velocity and acceleration at t=2 second.	A

3	Equation of motion of a moving particle is given by $s = t^3 - 5t^2 + 3t$, when will particle stop? Find the acceleration of particle at that time.	A
4	Equation of motion of a moving particle is given by $s = t^3 + 3t, t > 0$,when the velocity and acceleration will be equal?	A

5	Equation of motion of a moving particle is given by $s = t^3 - 6t^2 + 9t + 4$, when will particle change its direction? Find the s and a of particle at that time.	A
6	Find maximum and minimum of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$.	A

7	Find maximum and minimum of the function $f(x) = x^3 - 3x + 11$.	A
8	Find maximum and minimum of the function $f(x) = x^3 - 4x^2 + 5x + 7$.	A

9	Find maximum and minimum of the function $f(x) = 3x^3 - 4x^2 - x + 5$.	A
10	Find maximum and minimum of the function $f(x) = x\log_e x$	A

Answer Key:

Q-1: Answers

1)	**(c)**	2)	**(d)**	3)	**(a)**	4)	**(d)**	5)	**(a)**
6)	**(a)**	7)	**(b)**	8)	**(a)**	9)	**(b)**	10)	**(b)**

Q-2: Answers

1)	0, 30	2)	4,6
3)	$t=1/3$ or $t=3$ and $a_{t=1/3}=-8$ or $a_{t=3}=8$		$t=1$
5)	$t=1$ or $t=3$, $s_{t=1}=8$, $s_{t=3}=4$ and $a_{t=1}=-6$, $a_{t=3}=6$	6)	Maximum=38, Minimum=37
7)	Maximum=9, Minimum=13	8)	Maximum=9, Minimum=239/27
9)	Maximum=3, Minimum=1229/243	10)	Minimum = $\frac{-1}{e}$

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Differentiation and Its applications
1	https://bit.ly/2x9G8oe
2	https://bit.ly/3b5aTJV

Suggested Activities and website list for aspiring students

- https://www.whitman.edu/mathematics/calculus_online/chapter06.html

Date: ……………

Tutorial No. 8
(Unit No.3: Integration and its Applications)

Solve examples of integration using working rules, standard forms of integration and method of substitution.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of integration.

List of main formulas/working rules:

1	**Some standard formulas:**	
	1) $\int 1 \ dx = x + c$	2) $\int x^n \ dx = \frac{x^{n+1}}{n+1} + c$
	3) $\int \frac{1}{x} \ dx = log\lvert x\rvert + c$	4) $\int e^x \ dx = e^x + c$
	5) $\int a^x \ dx = \frac{a^x}{loga} + c$	6) $\int sinx \ dx = -cosx + c$
	7) $\int cosx \ dx = sinx + c$	8) $\int sec^2x \ dx = tanx + c$
	9) $\int cosec^2x \ dx = -cotx + c$	10) $\int secx \bullet tanx \ dx = secx + c$
	11) $\int tanx \ dx = log\lvert secx\rvert + c$	12) $\int \frac{f'(x)}{f(x)} \ dx = log\lvert f(x)\rvert + c$
	13) $\int cotx \ dx = log\lvert sinx\rvert + c$	14) $\int cosecx \bullet cotx \ dx = -cosecx + c$
	15) $\int secx \ dx = log\lvert secx + tanx\rvert + c$	16) $\int cosecx \ dx = log\lvert cosecx - cotx\rvert + c$
	17) $\int e^{kx} \ dx = \frac{e^{kx}}{k} + c$	18) $\int sinkx \ dx = \frac{-coskx}{k} + c$
	19) $\int \frac{1}{\sqrt{a^2-x^2}} \ dx = sin^{-1}\left(\frac{x}{a}\right) + c$	20) $\int \frac{1}{\sqrt{x^2 \pm a^2}} \ dx = log\lvert x + \sqrt{x^2 \pm a^2}\rvert + c$
	21) $\int \frac{1}{x^2+a^2} \ dx = \frac{1}{a} \bullet tan^{-1}\left(\frac{x}{a}\right) + c$	22) $\int \frac{1}{x^2-a^2} \ dx = \frac{1}{2a} \bullet log\left\lvert\frac{x-a}{x+a}\right\rvert + c$
	23) $\int \frac{1}{a^2-x^2} \ dx = \frac{1}{2a} \bullet log\left\lvert\frac{x+a}{x-a}\right\rvert + c$	24) $\int (f(x))^n \bullet f'(x) \ dx = \frac{(f(x))^{n+1}}{n+1} + c$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	$\int dx =$ ________				R
	(a) 0	(b) 1	(c) $x + c$	(d) $1 + c$	
2	$\int e^{-5x}\ dx =$ __________				U
	(a) $e^{-5x} + c$	(b) $-5x + c$	(c) $-5e^{-5x} + c$	(d) $-\frac{e^{-5x}}{5} + c$	
3	$\int 5x^4\ dx =$ __________ $+ c$				U
	(a) x^4	(b) $4x^3$	(c) $25x^5$	(d) x^5	
4	$\int tan^2x\ dx =$ __________ $+ c$				A
	(a) $2\ tanx\ sec^2x$	(b) $tanx + x$	(c) $tanx - x$	(d) $secx\ tanx$	
5	$\int \frac{1}{x^2+1}\ dx =$ _________				R
	(a) $sin^{-1}x + c$	(b) $tan^{-1}x + c$	(c) $cot^{-1}x + c$	(d) $-tan^{-1}x + c$	
6	$\int \frac{x}{x^2+1}\ dx =$ __________ $+ c$				A
	(a) $log\ (x^2 + 1)$	(b) $2\bullet log\ (x^2 + 1)$	(c) $\frac{1}{2}\bullet log\ (x^2 + 1)$	(d) $-2\bullet log\ (x^2 + 1)$	
7	$\int \frac{1}{\sqrt{x}}\ dx =$ __________ $+ c$				U
	(a) $log\sqrt{x}$	(b) $\frac{\sqrt{x}}{2}$	(c) $2\sqrt{x}$	(d) $\frac{1}{log\sqrt{x}}$	
8	$\int (sin^{-1}x + cos^{-1}x)\ dx =$ __________ $+ c$				A
	(a) $\frac{\pi}{2}$	(b) $\frac{\pi}{2}\bullet x$	(c) $\pi\bullet x$	(d) π	
9	$\int \frac{logx}{x}\ dx =$ __________ $+ c$				A
	(a) $logx$	(b) $\frac{1}{2}\ logx$	(c) $\frac{1}{2}(logx)^2$	(d) e^x	
10	$\int e^{-\log(secx)}\ dx =$ __________ $+ c$				A
	(a) $secx\ tanx$	(b) $cosx$	(c) $tanx$	(d) $sinx$	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

1	Evaluate: $\int \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 \, dx$	A
2	Evaluate: $\int \frac{x^2-5x-24}{x^2+3x}$ dx	A

3	Evaluate: $\int \frac{2+3sinx}{cos^2x}\,dx$	A
4	Evaluate: $\int (cosecx - cotx)\; cosecx\;\, dx$	A

5	Evaluate: $\int \frac{tanx}{secx + tanx}\, dx$	A
6	Evaluate: $\int \frac{12}{16 + 9x^2}\, dx$	A

7	Evaluate: $\int \frac{2x}{1+x^4}\, dx$	A
8	Evaluate: $\int \frac{1}{x \bullet logx}\, dx$	A

9	Evaluate: $\int \frac{sin\sqrt{x}}{\sqrt{x}}\ dx$	A
10	Evaluate: $\int tan^3x \bullet sec^2x\ dx$	A

11	Evaluate: $\int \frac{x^4 + x^2 + 1}{x^2 + 1}\, dx$	A
12	Evaluate: $\int sec^2x \bullet cosec^2x\ dx$	A

13	Evaluate: $\int \frac{cos2x}{sin^2x \bullet cos^2x}\ dx$	A
14	Evaluate: $\int cos5x \bullet sin3x\ dx$	A

15	Evaluate: $\int sin3x \bullet sinx \ dx$	A

Answer Key:

Q-1: Answers

1)	**(c)**	2)	**(d)**	3)	**(d)**	4)	**(c)**	5)	**(b)**
6)	**(c)**	7)	**(c)**	8)	**(b)**	9)	**(c)**	10)	**(d)**

Q-2: Answers

1)	$\frac{x^2}{2} + 2x + \log\lvert x\rvert + c$	2)	$x - 8\log\lvert x\rvert + c$
3)	$2tanx + 3secx + c$	4)	$cosecx - cotx + c$
5)	$secx - tanx + x + c$	6)	$tan^{-1}\left(\frac{3x}{4}\right) + c$
7)	$tan^{-1}(x^2) + c$	8)	$log(\log\lvert x\rvert) + c$
9)	$-2cos\sqrt{x} + c$	10)	$\frac{tan^4 x}{4} + c$
11)	$\frac{x^3}{3} + tan^{-1}x + c$	12)	$tanx - cotx + c$
13)	$-cotx - tanx + c$	14)	$\frac{cos2x}{4} - \frac{cos8x}{16} + c$
15)	$\frac{sin2x}{4} - \frac{sin4x}{8} + c$		

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Topic Name: Integration & Its application
1	Lecture – 1: https://www.youtube.com/watch?v=TQuO5cqIHKQ
2	Lecture – 2: https://www.youtube.com/watch?v=m8ZZtY8s4Ws
3	Lecture – 3: https://www.youtube.com/watch?v=yKpE8e2q_eU

Suggested Activities and website list for aspiring students

- https://www.mathsisfun.com/calculus/integration-introduction.html
- https://www.whitman.edu/mathematics/calculus_online/chapter08.html
- https://www.khanacademy.org
- https://www.accessengineeringlibrary.com/?implicit-login=true

Date: ……………

Tutorial No. 9
(Unit No. 3: Integration and its Applications)

Use the concept of integration by parts to solve related problems. Solve problems related to definite integral using properties.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of integration.

List of main formulas/working rules:

1	**Integration by parts:** $\int u \bullet v\ dx = u \int v\ dx - \int \left(\frac{d}{dx}(u) \bullet \int v\ dx\right) dx$	
2	**Some standard formulas:**	
	1) $\int_a^b f(x)\ dx = F(b) - F(a)$	2) $\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$
	3) $\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$	4) $\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$
	5) $\int_{-a}^a f(x)\ dx = 0\ ;\ If\ f(x)\ is\ odd$	6) $\int_{-a}^a f(x)\ dx = 2\int_0^a f(x)\ dx\ ; If\ f(x)\ is\ even$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	$\int x \bullet logx\ dx =$ ________		R
	(a) $logx \int x\, dx + \int \left(\frac{d}{dx}(x) \bullet \int logx\, dx\right) dx$	(b) $x \int logx\, dx - \int \left(\frac{d}{dx}(logx) \bullet \int x\, dx\right) dx$	
	(c) $logx \int x\, dx - \int \left(\frac{d}{dx}(logx) \bullet \int x\, dx\right) dx$	(d) $x \int logx\, dx + \int \left(\frac{d}{dx}(x) \bullet \int logx\, dx\right) dx$	
2	$\int_1^e \frac{1}{x}\ dx =$ ________		U

	(a) 1	(b) 0	(c) $log(e-1)$	(d) e	
3	$\int_{-1}^{1} 3x^2 - 2x + 1 \ dx = $ ________				U
	(a) 0	(b) 2	(c) 4	(d) 6	
4	$\int_{0}^{1} e^x \ dx = $ __________				U
	(a) e	(b) $e-1$	(c) $1-e$	(d) 1	
5	$\int_{0}^{\pi/6} 3 \bullet sin3x \ dx = $ _________				U
	(a) 3	(b) 0	(c) 1	(d) 9	
6	$\int_{0}^{\pi/2} cos2x \ dx = $ _________				U
	(a) π	(b) $\frac{\pi}{2}$	(c) $\frac{\pi}{4}$	(d) 0	
7	$\int_{-2}^{2} x - sinx \ dx = $ _________				U
	(a) 0	(b) $2 - sin2$	(c) $1 - sin2$	(d) 2	
8	$\int_{-\pi/2}^{\pi/2} sinx \ dx = $ ________				U
	(a) π	(b) 0	(c) $\frac{\pi}{4}$	(d) $-\pi$	
9	$\int_{-\pi/4}^{\pi/4} cos2x \ dx = $ ________				U
	(a) 0	(b) 1	(c) 2	(d) 4	
10	$\int_{-\pi}^{\pi} cos^2x \bullet sin^3x \ dx = $ _________				U
	(a) 2π	(b) 1	(c) 0	(d) -2π	

Q.2Do as directed (3 0R 4 MARKS QUESTIONS):

1	Evaluate: $\int x \bullet e^{-x}\ dx$	A
2	Evaluate: $\int x \bullet cosx\ dx$	A

3	Evaluate: $\int logx\ dx$	A
4	Evaluate: $\int_{2}^{3} 4x^3 - 2x + 6\ dx$	A

5	Evaluate: $\int_{0}^{1} \frac{x}{1+x}\, dx$	A
6	Evaluate: $\int_{1}^{3} \frac{2x}{1+x^2}\, dx$	A

7	Evaluate: $\int_{-\pi}^{\pi} tan^7 x \ dx$	A
8	Evaluate: $\int_{0}^{1} \frac{x^2 - 1}{x^2 + 1} \ dx$	A

9	Evaluate: $\int_{0}^{\pi/4} sec^2x \bullet e^{tanx}\ dx$	A
10	Evaluate: $\int_{0}^{\pi/4} \sqrt{1 + cos2x}\ dx$	A

11	Evaluate: $\int x^2 \bullet logx \ dx$	A
12	Evaluate: $\int_0^1 \frac{x^2}{1+x^6} \ dx$	A

13	Evaluate: $\int_0^{\pi/2} \frac{secx}{secx + cosecx}\,dx$	A
14	Evaluate: $\int_0^{\pi/2} \frac{\sqrt{sinx}}{\sqrt{cosx} + \sqrt{sinx}}\,dx$	A

15	Evaluate: $\int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x}+\sqrt{x}}\,dx$	A

Answer Key:

Q-1: Answers

1)	$(\boldsymbol{c})$	2)	$(\boldsymbol{a})$	3)	$(\boldsymbol{c})$	4)	$(\boldsymbol{b})$	5)	$(\boldsymbol{c})$
6)	$(\boldsymbol{d})$	7)	$(\boldsymbol{a})$	8)	$(\boldsymbol{b})$	9)	$(\boldsymbol{b})$	10)	$(\boldsymbol{c})$

Q-2: Answers

1)	$-e^{-x}(x+1)+c$	2)	$x\ sinx + cosx + c$
3)	$x\ (logx - 1) + c$	4)	66
5)	$1 - log2$	6)	$log5$
7)	0	8)	$1 - \frac{\pi}{2}$
9)	$e - 1$	10)	1
11)	$\frac{x^3}{3}\ logx - \frac{x^3}{9} + c$	12)	$\frac{\pi}{12}$
13)	$\frac{\pi}{4}$	14)	$\frac{\pi}{4}$
15)	1		

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Topic Name: Integration & Its application
1	Lecture – 1: https://www.youtube.com/watch?v=0z_MUp_pFJM
2	Lecture – 2:https://www.youtube.com/watch?v=R3pShXzXBMU

Suggested Activities and website list for aspiring students

- https://www.khanacademy.org
- https://www.geeksforgeeks.org/properties-of-definite-integrals/

Date: ……………

Tutorial No. 10
(Unit No. 3: Integration and its Applications)

Apply the concept of definite integration to find area and volume.

COURSE OUTCOME	Demonstrate the ability to solve engineering related problems based on applications of integration.

List of main formulas/working rules:

1	**Area:** Area of the region bounded between $y = f(x),\ X$ −axis, $x = a$line and$x = b$line is given by $\boldsymbol{Area\ (A)} = \int_a^b \boldsymbol{f(x)\ dx}$
2	**Volume:** Volume of a solid is formed by revolving curve about $X -$ axis $\mathbf{Volume\ (\ V\ )} = \int_a^b \boldsymbol{\pi y^2\ dx}$ Volume of a solid is formed by revolving curve about$Y$ −axis $\mathbf{Volume\ (\ V\ )} = \int_c^d \boldsymbol{\pi x^2\ dy}$

Q.1 Do as directed (3 0R 4 MARKS QUESTIONS):

1	Find area of the region bounded by the lines $y = 2x, x = 5$ and X −axis.	A
2	Find area enclosed by curve $y = x^2, X$ −axis $x = 1$ and $x = 2$	A

3	Find area of the region bounded by the curve $y = x^2$, line $x = 3, X$ −axis Y −axis.	A
4	Find area of the region bounded by the lines $x = 0, x = a, y = 0$ and $y = b$	A

5	Find area enclosed by curve $y = 3x^2$, line $x = 5$ and X −axis.	A
6	Find area of the region bounded by the curve $y = x^2, X$ −axis and $x = 2$	A

7	Find area of the region bounded by the curve $y = 3x^2, X$ −axis , $x = 2$ & $x = 3$	A
8	Find area of the region bounded by the curve $y = x^2, X$ −axis , $x = 2$ & $x = 3$	A

9	Find volume of a solid obtained by revolving area enclosed by the curve $y^2 = 2x$ and straight line $x = 3$ about X −axis	A
10	Find area of the region bounded by the curve $y^2 = x$ and straight line $x = 2$	A

11	Find area enclosed by the curve $y = x^2$ and straight line $x + y = 2$	A
12	Find area bounded by $X-, Y-$axis and straight line $x + y = 1$	A

13	Find the volume of a sphere of radius 1.	A
14	Find volume of a solid obtained by revolving area enclosed by $y^2 = 4ax$ and $x = a$ about X −axis	A

15	Find volume of a solid obtained by revolving area enclosed by straight lines $y = r, x = h, x = 0$ and $y = 0$ about X −axis	A

Answer Key:

Q-1: Answers

1)	25	2)	$\frac{7}{3}$
3)	9	4)	ab
5)	125	6)	$\frac{8}{3}$
7)	19	8)	$\frac{19}{3}$
9)	9π	10)	$\frac{8\sqrt{2}}{3}$
11)	$\frac{8}{3}$	12)	$\frac{1}{2}$
13)	$\frac{4\pi}{3}$	14)	$2\pi a^3$
15)	$\pi r^2 h$		

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Topic Name: Integration & Its application
1	Lecture – 1: https://www.youtube.com/watch?v=YfQySoZXcPE

Suggested Activities and website list for aspiring students

- https://www.khanacademy.org
- https://mathhints.com/applications-integration-area-volume/

Date: ……………

Tutorial No. 11
(Unit No. 4: Differential Equations)

Solve problems of the order, degree of differential equations and Variable Separable Method.

COURSE OUTCOME	Develop the ability to apply differential equations to significant applied problems.

List of main formulas/working rules:

1	**Order** The order of a differential equation is the order of the highest derivative that appears in the equation.
2	**Degree** The degree of a differential equation is the degree of the highest derivative that appears in the differential equation when all the derivatives appearing therein are free from radical signs and fractional powers.
3	**Variable Separable Form:** If a differential equation of first order and first degree can be reduced to the form $f(x)\,dx = g(y)\,dy$ then it is called separable equation.The general solution of above equation is given by $\int f(x)\ dx = \int g(y)\ dy + c$; $c =$Arbiter Constant

Q.1 Do as directed (ONE MARK QUESTIONS):

1	The order of a differential equation $\left(\frac{dy}{dx}\right)^2 + 2y = x$ is ________				R
	(a)0	(b)1	(c)2	(d)4	
2	The degree of a differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - y = 0$ is ________				R
	(a) 2	(b) 4	(c)1	(d)0	
3	The order of a differential equation $\frac{d^2y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$ is ________				R
	(a) 3	(b) 2	(c)1	(d)0	
4	The order and degree of $\left(\frac{d^4y}{dx^4}\right)^2 + \frac{d^3y}{dx^3}\frac{dy}{dx} - x^3\left(\frac{dy}{dx}\right)^5$ is ________respectively.				R
	(a) $2\ and\ 4$	(b)$4\ and\ 5$	(c)$5\ and\ 4$	(d)$4\ and\ 2$	
5	The order of a differential equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 - 2y = 0$ is ________				R
	(a) 1	(b) 2	(c)3	(d)4	
6	The degree of a differential equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 - 2y = 0$ is ________				R
	(a)1	(b) 2	(c)3	(d)4	
7	The order and degree of $x^2\, dy + y^2\, dx = 0$ is ________respectively				U
	(a)$1\ and\ 1$	(b) $1\ and\ 2$	(c)$2\ and\ 1$	(d)$2\ and\ 2$	
8	The order and degree of $e^{2y}\, dx + 3e^{5x-y}\, dy = 0$ is ________respectively				U
	(a) $2\ and\ 5$	(b) $5\ and\ 2$	(c)$1\ and\ 1$	(d)$1\ and\ 3$	
9	The degree of a differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/4} = \frac{d^2y}{dx^2}$ is ________				U
	(a)1	(b) 2	(c)3	(d)4	
10	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^5}$ is ________respectively				U
	(a)$2\ and\ 6$	(b)$2\ and\ 5$	(c)$3\ and\ 2$	(d)$5\ and\ 3$	

Q.2<u>Do as directed (3 0R 4 MARKS QUESTIONS):</u>

1	Solve: $2xy\,dx + x^2\,dy = 0$	A
2	Solve: $dy - 3x^2e^{-y}\,dx = 0$	A

3	Solve: $sinx \bullet cosy\ dy + siny \bullet cosx\ dx = 0$	A
4	Solve: $\frac{dy}{dx} + tanx \bullet tany = 0$	A

5	Solve: $(1 + x^3)dy - 3x^2y\,dx = 0$	A
6	Solve: $dy + 4xy^2\,dx = 0;\;\; y(0) = 1$	A

7	Solve: $x\,(1+y^2)\,dx + y\,(1+x^2)\,dy = 0$	A
8	Solve: $x\,dy + y\,dx = xy\,dy$	A

9	Solve:$e^y(1+x^2)\frac{dy}{dx}-2x\,(1+e^y)=0$	A
10	Solve: $tany\ dx + tanx \bullet sec^2y\ dy = 0$	A

Answer Key:

Q-1: Answers

1)	**(b)**	2)	**(c)**	3)	**(b)**	4)	**(d)**	5)	**(c)**
6)	**(d)**	7)	**(a)**	8)	**(c)**	9)	**(d)**	10)	**(a)**

Q-2: Answers

1)	$x^2y = c$	2)	$e^y = x^3 + c$
3)	$siny \bullet sinx = c$	4)	$siny = c \bullet cosx$
5)	$y = c\,(1 + x^3)$	6)	$y \bullet (2x^2 + 1) = 1$
7)	$(1 + y^2)(1 + x^2) = c$	8)	$y = log(xy) + c$
9)	$1 + e^y = c\,(1 + x^2)$	10)	$sinx\ tany = c$

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Topic Name: Differential Equations
1	Lecture – 1: https://www.youtube.com/watch?v=v6tGp_DLdRI
2	Lecture – 2: https://www.youtube.com/watch?v=PseIOjISdvI
3	Lecture – 3:https://www.youtube.com/watch?v=zSrUhIqeeGg

Suggested Activities and website list for aspiring students

- https://www.khanacademy.org

Date: ……………

Tutorial No. 12
(Unit No. 4: Differential Equations)

Apply the concept of linear differential equations to solve given differential equation. Explain the various applications of differential equations in engineering and real life.

COURSE OUTCOME	Develop the ability to apply differential equations to significant applied problems.

List of main formulas/working rules:

1	**Linear Differential Equation of First order (Leibnitz's Equation):** A differential equation is said to be linear if the dependent variable and its differential coefficients occur only in the first degree and are not multiplied together. It is commonly known as Leibnitz' s Linear Equation. Thus the standard form of a linear differential equation of first order is $\frac{dy}{dx} + Py = Q$ Where P and Q are functions of x.
2	**Integrating Factor (I.F.)**$= \boldsymbol{e^{\int P\, dx}}$
3	The solution of above differential equation is given by $\boldsymbol{y\,(I.F.) = \int Q \bullet (I.F.)\, dx + c}$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	Which one of the following is a linear differential equation of first order				R
	(a) $\frac{dy}{dx} + x\,siny = e^y$	(b) $\frac{dy}{dx} + y^2 = \sqrt{x+y}$	(c) $\frac{dy}{dx} - 2y = e^x$	(d) $\left(\frac{dy}{dx}\right)^2 + xy = \sqrt{x}$	
2	Which one of the following is not a Leibnitz′s differential linear equation				R
	(a) $\frac{dy}{dx} + 3y = \frac{1}{2}$	(b) $\frac{dy}{dx} + \frac{x}{y} = logy$	(c) $\frac{dy}{dx} - y = e^x$	(d) $\frac{dy}{dx} + \frac{y}{x} = logx$	
3	Which one of the following is a Leibnitz′s differential linear equation				U
	(a) $x\,dy - y\,dx = 0$		(b) $\sqrt{1+y^2}\,dx - \sin(x+y)\,dy = 0$		
	(c) $(x+y)\,dy = (x-y)\,dx$		(d) $2xy\,dy = (x^2+y^2)\,dx$		
4	The integrating factor of $\frac{dy}{dx} + \frac{y}{x} = logx$ is ________				U
	(a) $x \bullet logx$	(b) x	(c) $logx$	(d) x^2	
5	The integrating factor of $\frac{dy}{dx} - y\,tanx = log(sinx)$ is ________				U
	(a) $cosx$	(b) $sinx$	(c) $tanx$	(d) $log(sinx)$	
6	The integrating factor of $\frac{dy}{dx} + 3y = x$ is ________				U
	(a) $3x$	(b) e^x	(c) e^{2x}	(d) e^{3x}	
7	The integrating factor of $\frac{dy}{dx} + y = 3x$ is ________				U
	(a) e^x	(b) $logx$	(c) 3	(d) x	
8	The integrating factor of $\frac{dy}{dx} + \frac{y}{x+1} = \frac{1}{(x+1)^2}$ is ________				A
	(a) $x+1$	(b) $log\,\lvert x+1\rvert$	(c) $(x+1)^2$	(d) e^{x+1}	
9	The integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is ________				U
	(a) $logx$	(b) x^2	(c) $logx^2$	(d) $None\ of\ these$	
10	The general solution of $\frac{dy}{dx} + y = 0$ is ________				A
	(a) $ye^x = c$	(b) $ye^x = x + c$	(c) $xy = c$	(d) $y = cx$	

Q.2 <u>Do as directed (3 0R 4 MARKS QUESTIONS):</u>

1	Solve: $\frac{dy}{dx} + 2y = 3e^{x}$	A
2	Solve: $\frac{dy}{dx} + 2xy = e^{-x^2}$	A

3	Solve: $\frac{dy}{dx} - \frac{3y}{x} = x^3$	A
4	Solve: $\frac{dy}{dx} - y\ cotx = 2x\ sinx$	A

5	Solve: $\frac{dy}{dx} + y\ tanx = cos^2x$	A
6	Solve: $x \bullet logx\ \frac{dy}{dx} + y = x^2$	A

7	Solve: $\frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$	A
8	Solve: $(x^2+1)\frac{dy}{dx} + 2xy = 3x^2$	A

9	Solve: $(1+x^2)\frac{dy}{dx}+2xy=cosx$	A
10	Solve: $cosx\ \frac{dy}{dx}+y=sinx$	A

Answer Key:

Q-1: Answers

1)	**(c)**	2)	**(b)**	3)	**(a)**	4)	**(b)**	5)	**(a)**
6)	**(d)**	7)	**(a)**	8)	**(a)**	9)	**(d)**	10)	**(a)**

Q-2: Answers

1)	$ye^{2x} = e^{3x} + c$	2)	$ye^{x^2} = x + c$
3)	$y = x^3(x + c)$	4)	$y \bullet cosecx = x^2 + c$
5)	$y \bullet secx = sinx + c$	6)	$y \bullet logx = \frac{x^2}{2} + c$
7)	$y = (x + 1)(e^x + c)$	8)	$y\,(x^2 + 1) = x^3 + c$
9)	$y\,(1 + x^2) = sinx + c$	10)	$y\,(secx + tanx) = secx + tanx - x + c$

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ **(Link: https://www.youtube.com/@dteguj8385)** (Directorate of Technical Education Department, Government of Gujarat)
	Topic Name: Differential Equations
1	Lecture – 1: https://www.youtube.com/watch?v=SbPbGTbS8y0
2	Lecture – 2: https://www.youtube.com/watch?v=zSrUhIqeeGg

Suggested Activities and website list for aspiring students

- https://www.khanacademy.org

Date: ……………

Tutorial No. 13
(Unit No. 5: Statistics)

Solve examples of Mean for the given data.

COURSE OUTCOME	Solve applied problems using the concept of mean.

List of main formulas/working rules:

Mean $\overline{x}$

Ungroup data

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$x_1, x_2, ..., x_n$ are observations

$n =$ total number of observations

Grouped data

Discrete variable

(1) $\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{n}$ $\quad n = \sum_{i=1}^{n} f_i$

(2) $\overline{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{n}$

A = assumed mean

$n = \sum fi$, $d_i = x_i - A$

Continuous variable

Discrete method

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{n} \quad n = \sum_{i=1}^{n} f_i$$

x_i = mid value of class

Shortcut method

$$\overline{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{n} \times c$$

x_i = mid value of class

c = class length

A = assume mean

x_i whose f_i is maximum

$$d_i = \frac{x_i - A}{c}$$

Q.1 <u>Do as directed (ONE MARK QUESTIONS):</u>

1	Find the mean for the first five prime numbers.				A
	(a) 3.6	(b) 5.6	(c) 5.4	(d) 5.2	
2	Find the mean for the first ten natural numbers.				A
	(a) 5.5	(b) 5	(c) 5.4	(d) 6	
3	Find the mean for the following data. 32, 26, 41, 35, 28, 42, 36, 40, 33, 42				U
	(a) 35	(b) 34.5	(c) 35.4	(d) 35.5	
4	Find the mean for the first five natural numbers.				A
	(a) 3	(b) 2.8	(c) 2.5	(d) 2.6	
5	Find the mean for the first ten even numbers.				A
	(a) 10.8	(b) 55	(c) 11	(d) 11.2	
6	If the mean of 5 observations $x-8,\ x-5,\ x-3,\ x+2,\ x+4$ is 8 then find the value of *x*.				U
	(a) 10	(b)12.4	(c) 6	(d) 3.6	
7	The range of the data 17, 15, 25, 34, 32 is ______				U
	(a) 32	(b) 17	(c) 15	(d) 19	
8	The mean for grouped data with *n* observations is ______				R
	(a) $\overline{x}=\frac{x_i f_i}{n}$	(b) $\overline{x}=\frac{\sum_{i=1}^{n} x_i f_i}{n}$	(c) $\overline{x}=\sum_{i=1}^{n} x_i f_i$	(d) $\overline{x}=x_i f_i$	
9	What is the class length for the following data. Class: 0-5, 6-11, 12-17, 18-23, 24-29, 30-35 Frequency: 8, 3, 1, 6, 2, 4				U
	(a) 4	(b) 6	(c) 5	(d) 7	
10	The mean for grouped data with *n* observations is ______				R
	(a) $\overline{x}=\frac{\sum_{i=1}^{n} f_i d_i}{n}\times c$	(b) $\overline{x}=A+\sum_{i=1}^{n} x_i f_i\times c$	(c) $\overline{x}=A+\frac{\sum_{i=1}^{n} f_i d_i}{n}\times c$	(d) $\overline{x}=A+\frac{\sum_{i=1}^{n} f_i d_i}{n}$	

Q.2Do as directed (3 0R 4 MARKS QUESTIONS):

1	Marks obtained by 10 students in Mathematics are 33, 26, 44, 35, 28, 42, 36, 40, x and 42. If mean of the data is 35.90, then find the x .	A
2	The dividend declared by 60 different companies in the year 2020-21 are show in the following table. Find the mean of the data.	A

Dividend in (%)	10	12	15	18	20	22	25
No. of companies	7	10	12	6	12	8	5

3	Find the mean value for the following group data.	A

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

4	If mean is 19 for the following data then find the missing frequency *a*.	A

x_i	6	10	14	18	24	28	30
f_i	2	4	7	a	8	4	3

5 Find the mean for the following group data. A

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	*15*	8	3	2

6 The frequency distribution of age of 60 staffs of college is as below. Find the mean of given data. A

Age	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
No. of staff	5	7	9	11	10	8	6	4

7	The weight distribution of the students are given below. Find the mean weight.	A

Weight	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 -60
No. of students	5	10	25	30	20	10

8	Determine the mean for the following group data.	A

x_i	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
f_i	12	13	22	11	10	8	8	1

9	The mean of the following frequency distribution of 100 observations is 148. Find the missing frequencies x and y.	A
10	If mean of 25 observations is 50 and mean of other 75 observations is 60. Considering all the observation then find the mean.	A

Class	0-49	50-99	100-149	150-199	200-249	250-299	300-349
Frequency	10	15	x	20	15	y	2

Answer Key:

Q-1: Answers

1)	**(b)**	2)	**(a)**	3)	**(d)**	4)	**(a)**	5)	**(c)**
6)	**(a)**	7)	**(d)**	8)	**(b)**	9)	**(b)**	10)	**(c)**

Q-2: Answers

1)	33	2)	16.98
3)	100	4)	12
5)	62	6)	38.83
7)	33	8)	3.82
9)	$x = 30$, $y = 8$	10)	57.50

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ https://youtu.be/ANvqwNfMMKs (Directorate of Technical Education Department, Government of Gujarat)
1	**Online Calculator to calculate Mean** https://atozmath.com/StatsUG.aspx?q=1 https://atozmath.com/StatsG.aspx?q=1

Date: ……………

Tutorial No. 14
(Unit No. 5: Statistics)

Solve examples of Mean deviation and Standard deviation for the given data.

COURSE OUTCOME	Solve applied problems using the concept of mean.

<u>List of main formulas/working rules:</u>

Mean deviation $\delta\overline{x}$

Ungroup data

$$\delta\overline{x} = \frac{1}{n}\sum\left|x_i - \overline{x}\right|, \qquad \overline{x} = \frac{\sum x_i}{n}$$

$x_1, x_2, \ldots, x_n$ are observations

$n =$ total number of observations

Grouped data

Discrete variable

$$\delta\overline{x} = \frac{1}{n}\sum f_i\left|x_i - \overline{x}\right|,$$

$$n = \sum f_i, \quad \overline{x} = \frac{\sum f_i x_i}{n}$$

Continuous variable

$$\delta\overline{x} = \frac{1}{n}\sum f_i\left|x_i - \overline{x}\right|,$$

$$\overline{x} = A + \frac{\sum f_i d_i}{n} \times c$$

$$n = \sum f_i, \quad d_i = \frac{x_i - A}{c}$$

$c =$ class length

Standard deviation ***S***

Ungroup data

$$S=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n}},\quad \overline{x}=\frac{\sum x_i}{n}$$

OR

$$S=\sqrt{\frac{\sum x_i^{\,2}}{n}-\left(\frac{\sum x_i}{n}\right)^2}$$

OR

$$S=\sqrt{\frac{\sum d_i^{\,2}}{n}-\left(\frac{\sum d_i}{n}\right)^2}$$

A = Assume mean and

$$d_i=x_i-A$$

Grouped data

Discrete Variable

$$S=\sqrt{\frac{\sum f_i\left(x_i-\overline{x}\right)^2}{n}}$$

$$n=\sum f_i \qquad \overline{x}=\frac{\sum f_i x_i}{n}$$

OR

$$S=\sqrt{\frac{\sum f_i x_i^{\,2}}{n}-\left(\frac{\sum f_i x_i}{n}\right)^2}$$

OR

$$S=\sqrt{\frac{\sum f_i d_i^{\,2}}{n}-\left(\frac{\sum f_i d_i}{n}\right)^2}$$

A = Assume mean and

$$d_i=x_i-A$$

Continuous Variable

$$S=\sqrt{\frac{\sum f_i\left(x_i-\overline{x}\right)^2}{n}}$$

$x_i=$ mid value of class

$$n=\sum f_i \qquad \overline{x}=\frac{\sum f_i x_i}{n}$$

OR

$$S=\sqrt{\frac{\sum f_i x_i^{\,2}}{n}-\left(\frac{\sum f_i x_i}{n}\right)^2}$$

OR

$$S=\sqrt{\frac{\sum d_i^{\,2}}{n}-\left(\frac{\sum d_i}{n}\right)^2}\times c$$

A = Assume mean and

$$d_i=\frac{x_i-A}{c},\ c=\text{class length}$$

Q.1 Do as directed (ONE MARK QUESTIONS):

1	The standard deviation for assumed mean *A* of *n* observation				R
	(a) $S=\sqrt{\frac{\sum f_i\left(x_i-\overline{x}\right)^2}{n}}$	(b) $S=\sqrt{\frac{\sum f_i x_i^2}{n}-\left(\frac{\sum f_i x_i}{n}\right)^2}$	(c) $S=\sqrt{\frac{\sum f_i d_i^2}{n}-\left(\frac{\sum f_i d_i}{n}\right)^2}$	(d) none of these	
2	The mean deviation of *n* observation is				R
	(a) $\delta\overline{x}=\sum\left\|x_i-\overline{x}\right\|$	(b) $\delta\overline{x}=\frac{\left\|x_i-\overline{x}\right\|}{n}$	(c) $\delta\overline{x}=\sum\left(\frac{\left\|x_i-\overline{x}\right\|}{n}\right)$	(d) $\delta\overline{x}=\frac{1}{n}\sum\left\|x_i-\overline{x}\right\|$	
3	The mean deviation for continuous variable and grouped data of *n* observation is				R
	(a) $\delta\overline{x}=\frac{1}{n}\sum f_i\left\|x_i-\overline{x}\right\|$	(b) $\delta\overline{x}=\frac{1}{n}\sum f_i\left\|x_i-\overline{x}\right\|\times c$	(c) $\delta\overline{x}=\frac{1}{n}\sum\left\|x_i-\overline{x}\right\|\times c$	(d) $\delta\overline{x}=\frac{1}{n}\sum\left\|x_i-\overline{x}\right\|$	
4	The standard deviation of grouped data is				R
	(a) $S=\frac{\sqrt{\sum f_i\left(x_i-\overline{x}\right)^2}}{n}$	(b) $S=\sqrt{\frac{\sum f_i\left(x_i-\overline{x}\right)^2}{n}}$	(c) $S=\sqrt{\sum f_i\left(x_i-\overline{x}\right)^2}$	(d) $S=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n}}$	

Q.2 Do as directed (3 0R 4 MARKS QUESTIONS):

<table>
<tr><td>1</td><td>Find the mean deviation for the ungrouped data. 4, 6, 2, 4, 5, 4, 4, 5, 3, 4</td><td>A</td></tr>
<tr><td>2</td><td>Find the mean deviation for the following frequency distribution.
<table><tr><td>x_i</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>f_i</td><td>4</td><td>9</td><td>10</td><td>8</td><td>6</td><td>3</td></tr></table></td><td>A</td></tr>
</table>

3	Calculate the mean deviation of the data: <table><tr><td>Class</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td></tr><tr><td>Frequency</td><td>15</td><td>18</td><td>21</td><td>17</td><td>12</td></tr></table>	A
4	Find the standard deviation for the following ungrouped data. 6, 7, 10, 12, 13, 4, 8, 12	A

5	Calculate the standard deviation for the following ungrouped data. 120, 132, 148, 136, 142, 140, 165, 153	A
6	Find the standard deviation for the following discrete grouped data.	A

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

7	Calculate the standard deviation for the following continuous grouped data. Class: 0-10, 10-20, 20-30, 30-40, 40-50 Frequency: 5, 8, 15, 16, 6	A
8	The crushing strength of 45 cement concrete blocks are recorded as then find the standard deviation. Crushing strength in kg/cm^2: 145-155, 155-165, 165-175, 175-185, 185-195, 195-205 No. of blocks: 6, 7, 9, 14, 4, 5	A

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Crushing strength in kg/cm^2	145-155	155-165	165-175	175-185	185-195	195-205
No. of blocks	6	7	9	14	4	5

9	Find the mean deviation for the following frequency distribution. <table><tr><td>Monthly wages</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>Workers</td><td>1</td><td>3</td><td>7</td><td>5</td><td>2</td><td>2</td></tr></table>	A
10	Weekly expenditure of students are given below. Find the standard deviation of the data. <table><tr><td>Weekly expenditure below Rs.</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>No. of students</td><td>6</td><td>16</td><td>28</td><td>38</td><td>46</td></tr></table>	A

Answer Key:

Q-1: Answers

1)	**(c)**	2)	**(d)**	3)	**(a)**	4)	**(b)**

Q-2: Answers

1)	0.74	2)	1.195
3)	10.895	4)	3.04
5)	12.80	6)	6.77
7)	11.49	8)	14.817
9)	1.05	10)	5.86

Link of BISAG Lectures

	YouTube Channel name: DTEGUJ https://youtu.be/vXp5YMyQOd8 (Directorate of Technical Education Department, Government of Gujarat)
1	**Online calculator to calculate standard deviation** https://atozmath.com/StatsUG.aspx?q=4 https://atozmath.com/StatsUG.aspx?q=8 https://atozmath.com/StatsG.aspx?q=3 https://atozmath.com/StatsG.aspx?q=8

Applied Mathematics
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